PROPAGATION OF SMALL DISTURBANCES IN
A NON-EQUILIBRIUM GAS-SOLID FLOW

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Abstract:

The velocity of propagation of small amplitude, steep fronted pressure disturbances (the velocity of sound) in a non- equilibrium gas-solid suspension flow has been investigated theoretically. A well established approach to the theory of compressible gas solids flow is to adopt the equations for the one - dimensional flow of a compressible clean gas to a two-phase mixture. The equations are derived, taking the particle size distribution into account, in the absence of several assumptions which usually used by previous investigators, such as ; the solid particles and the gas are in equilibrium and neglecting the solids volume. The velocity of sound is shown to depend on the proportion by volume of solid particles in the mixture, the particle velocity lag, the particle material density, the direction of wave propagation relative to the flow direction and the amount of solid particles in the mixture. The results were checked against the experimental results,obtained by previous investigators, and a good agreement has been obtained.

## 1. Introduction:

Gas-solid flows assume to be important in several engineering problems as flow in rockets, nuclear reactors, fuel sprays, pulverised fuel fed boilers , air pollution etc. Also, with the adrancement of space technology, the dynamics of fluid-particle systems has found applications in such extraterrestrial fields as lunar ash flow and predictably in the studies of other planets , PAI and Hesieh [1].

The Mach number concept is a very useful one in studies of compressible flows. In order to make use of this in two-phase flow analysis, it is necessary to have an expression for the sonic velocity in the mixture. various attempts have been made to drive such an expression for the speed of propagation of a small amplitude pressure disturbance in both chemically reacting gases and two-phase flows. In chemically reacting gases, Hoglund [2] compared the flow with that of a chemically reacting gas, where two sound velocities can be defined, the frozen sound velocity, associated with zero reaction rates, which is associated with the disturbance propagating in the gas alone, and the equilibrium sound velocity where the reaction rates are so high that chemical equilibrium is reached every - where, which for a two-phase flow would be the equilibrium case in which there are no velocity or temperature differences between the phases. Kliegel [3] , for a gas solid flow, defined two speeds of sound related to the response rate of the solid particles to the passage of a small amplitude pressure disturbance:

(a) The frozen speed of sound where the particles are unaffected by the pressure wave, and

(b) The equilibrium speed of sound where the particles remain in kinetic and thermal equilibrium with the gas throughout.

Kliegel also suggested that an unlimited number of possibilities existed between these two extremes.

He also neglected the solids volume in comparison with the gas volume. Rudinger [4] has derived an equilibrium speed of sound , taking the solids volume into account. Because the equilibrium case is not found in practice, some investigators have tried to take account of the lags between the phases. Kliegel [3], In his paper, considered in his analysis of flow through a convergent nozzle, the velocity and temperature lags are constant over the step length. In his analysis, also he defined a flow Mach number based on a constant fractional lag definition of the sonic velocity. Mobbs, et.al[5], in a comprehensive analysis of gas-solids flow, used the concept of constant fractional lag, which defined by Kliegel [3]. They also attempted to account for the responce of the solid particles to the passage of the disturbance by a constant fractional lag assumption. It was assumed that the ratio of the solids velocity to the gas velocity would be the same before and after the wave.

In the present paper, a mathematical model for the speed of propagation of a small amplitude pressure disturbance in a non-equilibrium gas-solid flow is presented here, the effects of solids loading ratio, solids volume, the particle material density and the velocity lag on the sound velocity have been considered. The powders used as a test particles, in the present investigation (polystyrene , glass spheres, silica and steel shot) are polydisperse powders and the particle size disturbution has been taken into account. These powders were chosen as an alternative solid phase to establish the effect of particle material density on the speed of sound.

Nomenclature:

- $a_g$ : Gas velocity of sound.
- $a_m$ : Gas-solids mixture speed of sound, velocity of propagation of a small amplitude pressure wave in the mixture.
- $C_D$ : Particle drag coefficient
- $C_s$ : Specific heat of the solid particles
- $C_v$ : Gas specific heat at constant volume.
- D : Particle diameter.
- K : Particle velocity lag = Ratio of solids velocity to gas velocity.
- $K_g$ : Thermal conductivity of the gas
- $m_g$ : Mass rate of flow of the gas
- $m_s$ : Mass rate of flow of the solids
- ns : Number of particle sizes considered
- $N_u$ : Particle Nusselt number.
- P : Static pressure
- $P_r$ : Prandtl number
- R : Gas constant
- t : Time
- $T_g$ : Gas temperature
- $T_s$ : Solids temperature
- u : Gas velocity
- V : Solid particle velocity
- x : Distance along the duct
- X : Ratio of mass rate of flow of solids to mass rate of flow of gas.
- $y$ : Proportion of solids by volume.
- $\rho_g$ : Gas density
- $\rho_p$ : Solid particle density
- $\rho_s$ : Distributed solids density (based on gas volume)
- $\gamma$ : Ratio of gas specific heats
- $\mu$ : Gas dynamic viscosity.

Subscript:

- i : ith particle size.

## 2. Analysis:

### 2.1- Assumptions:

The following assumptions apply to all the analytical work described below:

1- Flows are one-dimensional continuum flows and steady.

2- The density and specific heat of the particle material are constant.

3- The gas is considered to be a perfect gas with constant specific heats.

4- The particles are electrically inert, and do not interact kinetically; thermal motion of the particles does not contribute to the pressure of the system.

5- The solid particles are of uniform size and shape.

6- There is no mass transfer between the gas and the particles.

### 2.2- Basic Equations:

Consider a stationary, infinitesimal pressure discontinuity, propagating at a relative velocity equal to the downstream gas velocity in a constant area duct, Fig.(1). A head of the wave , the solid particles are assumed to be in temperature and velocity disequilibrium with the gas. The basic equations governing the motion are:

The gas continuity equation

$$\rho_g u = (u + du)(\rho_g + d\rho_g)$$

or,

$$\rho_g du + u d\rho_g = 0 \qquad \ldots\ldots(1)$$

The solids continuity equation

$$\rho_{si}\, dv_i + V_i\, d\rho_{si} = 0 \qquad \ldots\ldots(2)$$

The overall momentum equation:

$$\rho_g\, udu + \sum_{i=1}^{ns} (\rho_{si} . V_i . dv_i) + dP = 0$$

or

$$\rho_g\, udu + \sum_{i=1}^{ns} (X_i\, u\, \rho_g\, dV_i) + dP = 0 \qquad \ldots\ldots(3)$$

Where,

$$X_i = \frac{m_{si}}{m_g} = \frac{V_i\, \rho_{si}}{u . \rho_g} \qquad \ldots\ldots(4)$$

From equations (1) and (3) and putting $u = a_m$

$$\therefore a_m^2 = \frac{dP}{d\rho_g} \cdot \left[ \frac{1}{1 + \sum_{i=1}^{ns} (X_i \frac{dV_i}{du})} \right] \quad ......(5)$$

The energy equation:

To determine the value of $\frac{dP}{d\rho_g}$, it is necessary to to use the energy equation which takes the form.

$$udu + C_v dT_g + d(\frac{P}{\rho_g}) + \sum_{i=1}^{ns} \left[ X_i (V_i dV_i + C_s d\, T_{si} + \frac{dP}{\rho_p}) \right] = 0$$

From the equation of state; $dT_g = \frac{1}{R} d(\frac{P}{\rho_g})$; and $C_v = \frac{R}{\gamma - 1}$;
the energy equation may be written as:

$$\left[ u + \sum_{i=1}^{ns} (X_i\, V_i \frac{dV_i}{du}) \right] du + \left[ (\frac{R}{\gamma-1} + \sum_{i=1}^{ns} (X_i C_s \frac{dT_{si}}{dT_g} )) \frac{1}{R} + 1 \right] d(\frac{P}{\rho_g}) + \sum_{i=1}^{ns} (K_i\, y_i) \frac{dP}{\rho_g} = 0 \quad .....(6)$$

Where;

$$K_i = V_i/u \quad , \text{ and } \quad y_i = (\rho_g X_i)/(\rho_p K_i) \quad .......(7)$$

The overall momentum equation, Equ.(3) may be written, after considering the solids volume as follows:

$$udu = -\frac{dP}{\rho_g} \left[ \frac{1 + \sum_{i=1}^{ns} y_i}{1 + \sum_{i=1}^{ns} (X_i \frac{dV_i}{du})} \right] \quad .......(8)$$

$$\therefore d(\frac{P}{\rho_g}) = (\rho_g\, dP - P\, d\rho_g)/\rho_g^2 \quad .......(9)$$

Substitute Eqs(8) and (9) into (6),

$$\therefore \frac{dP}{d\rho_g} = \frac{R\, T_g}{1 - \frac{Z_2}{Z_1}\left[ 1 + \sum_{i=1}^{ns} (X_i K_i \frac{dV_i}{du}) \right] + \frac{Z_3}{Z_1}} \quad ......(10)$$

Where;

$$Z_1 = \left[ \left( \frac{R}{\gamma - 1} + \sum_{i=1}^{ns} (X_i\ C_s\ \frac{dT_{si}}{dT_g}) \right] \frac{1}{R} + 1$$

$$Z_2 = \frac{1 + \sum_{i=1}^{ns} y_i}{1 + \sum_{i=1}^{ns} (X_i\ \frac{dV_i}{du})}$$

$$Z_3 = \sum_{i=1}^{ns} (K_i\ y_i)$$

Particle momentum equation:

A single particle moving in a gas stream is subjected to a force due to the pressure gradient in the surrounding gas in addition to viscous drag. The equation of motion for a particle may therefore be written as (Rudinger [6]).

$$\frac{\pi D^3}{6} \rho_p\ V_i\ \frac{dV_i}{dx} = \frac{1}{2}\ C_D\ \rho_g\ (u - V_i) \left| u - V_i \right| \frac{\pi}{4} D^2 - \frac{\pi D^3}{6} . \frac{dP}{dx} \quad ..(11)$$

In most cases of gas-solid flow the pressure gradient term is negligible compared with the viscous drag term. However, In our case, when a particle passes through a steep fronted pressure wave, the pressure gradient term in Equ. (11) will predominate over the viscous drag term. According to this assumption, Equ.(11) reduces to ;

$$V_i\ \frac{dV_i}{dx} = - \frac{1}{\rho_p} . \frac{dP}{dx}$$

The particle velocity change is given by ;

$$dV_i = - \frac{1}{V_i} . \frac{1}{\rho_p}\ dP \quad .......(12)$$

Conbining Equs(8) and (12);

$$\therefore \frac{dV_i}{du} = \frac{1}{\sum_{i=1}^{ns} \left[ \frac{\rho_p}{\rho_g} (K_i\ (1 + y_i)) - X_i \right]} \quad .......(13)$$

Particle Heat Transfer Equation:

The particle heat transfer equation may be written as, [6] ;

$$\frac{dT_{si}}{dt} = \frac{1}{\tau_t}(T_g - T_{si}) \qquad \text{......(14)}$$

Where,

$\tau_t$ = is the thermal relaxation time

$$= \rho_p C_s D^2/6K_g N_u \qquad \text{......(15)}$$

For a fluid particle subjected only to viscous drag and whose Reynolds number is sufficiently low for stokes law to apply , the equation of motion takes the following form ;

$$\frac{dV_i}{dt} = \frac{1}{\tau_v}(u - V_i) \qquad \text{......(16)}$$

Where,

$\tau_v$ = is the viscous drag relaxation time

$$= \frac{\rho_p D^2}{18\mu} \qquad \text{......(17)}$$

Comparing these two relaxation times, Equs(15)and (17),

$$\therefore \frac{\tau_t}{\tau_v} = \frac{3C_s\mu}{K_g . N_u} = \frac{3P_r}{N_u}$$

In most previous studies of gas-solid flow; for example in [3 - 6] ; Nu = 2 at low Reynolds numbers, and $P_r$ is of order 2/3 for most gases, Therefore, $\tau_t/\tau_v \simeq 1$ , i.e. the thermal relaxation time and the viscous drag relaxation time are of the same order. This leads to the conclusion that if viscous drag has a negligible effect in producing a change in particle velocity, it may also be assumed that the change in particle temperature is negligible, (Warda[7]), i.e.;

$$\frac{d\,T_{si}}{d\,T_g} = 0 \qquad \text{......(18)}$$

Substitute Equs(13), ~~(18) into~~ (10), the velocity of sound becomes :

$$a_m = \pm \left\{ \frac{RT_g\left[1 - \frac{\rho_g}{\rho_p}\sum_{i=1}^{ns}\left(\frac{X_i}{K_i}\right) / \left(1 + \sum_{i=1}^{ns} y_i\right)\right]}{1 - \frac{\gamma-1}{\gamma}\left[1 + \sum_{i=1}^{ns}\left(y_i(1-K_i)\right)\right] + \frac{\gamma-1}{\gamma}\frac{\rho_g}{\rho_p}\sum_{i=1}^{ns}\left(\frac{X_i}{K_i}\right)} \right\}^{\frac{1}{2}} \qquad .(19)$$

The plus and minus signs in the above equation are for the wave travelling upstream and downstream, respectively, i.e. the velocity of propagation varies with the direction of propagation relative to the flow direction and also depends on the value of solids loading ratio (X), velocity lag (K), gas/solid density ratio ($\rho_g/\rho_p$) and the solids volume (y). From Equ(19) it is clear that, if the wave propagation in a clean gas, the value of $a_m$ reduces to that for the gas phase only, i.e for $X = 0.0$ , $a_m = a_g = \sqrt{\gamma R T_g}$ .

For the case of (y = 0) , i.e. the solids volume is neglected, Equ (19) reduces to ;

$$a_m = \pm \left[ \frac{R\, T_g \left(1 - \frac{\rho_g}{\rho_p} \sum_{i=1}^{ns} \frac{X_i}{K_i}\right)}{1 - \frac{\gamma - 1}{\gamma} \left(1 + \frac{\rho_g}{\rho_p} \sum_{i=1}^{ns} \frac{X_i}{K_i}\right)} \right]^{\frac{1}{2}} \qquad \ldots\ldots(20)$$

3. Results and Discussion:

In the calculation described below, the following constants are assumed throughout :

1- For the gas phase;
Air is used as the gas phase with the following properties :

Gas constant (R) = 287 J/kg.k°
Ratio of specific heats(K) = 1.4
Speed of sound ($a_g$) = 340 m/s.

2 2- For the solid phase:

| Powder | $\rho_p$ ,(Kg/m$^3$) | $\rho_g/\rho_p$ |
|---|---|---|
| Polystyrene | 1060 | 0.003 |
| Glass spheres | 2450 | $1.29 \times 10^{-3}$ |
| Steel shot | 7770 | $4.09 \times 10^{-4}$ |
| Silica | 1202 | $2.64 \times 10^{-3}$ |

3.1- Effect of velocity lag (K) on the speed of propagation.

Figs(2-5) indicate the variation of the theoretical mixture speed of sound, as calculated from Equ (19) for different values of solids loading ratio (X) and for different type of powder materials, with the velocity lag (K). In order to study the effect of considering the solids volume on the variation of mixture speed of sound, the calculations were made using Equ (19), where the effect of solids volume on the calculation is considered. Solid curves, which are presented in all figures, indicate this situation, while the broken curves indicate the case where the solids volume is neglected. A comparison between the results in which the solids volume was considered in the calculation and the results where the solids volume was neglected, illustrates that, the mixture speed of sound is affected by the solids volume. The velocity of propagation has a smaller value, for the case of neglecting the solids volume, than that in the case of considering the solids volume This conclusion was also reached by Warda[7] in his analysis of a steady normal shock wave and by Ibrahim[8] in his analysis of unsteady normal shock wave. They found that the rate at which the velocity falls increases if the volume of particles is neglected. Varma and Chopra [9], also found that the gas velocity is higher in the relaxation zone, behind the shock wave, as the solids volume is increased. The present investigation shows that this conclusion also applies in the analysis of the propagation of small pressure disturbances in a non-equilibrium suspension flow. For a high solids loading ratio; a comparison can be made; for the case of $X = 10$, it is shown that some errors are encountered due to neglecting the solids volume ($y$). This is because the amount of solids is increased due to an increase in the solids loading ratio at the same value of gas - particle density ratio. These errors will be small if the gas-particle density ratio is also high.

From these figures it can also be seen that the mixture speed of sound increases with increasing particle velocity lag (K). This is attributed to less interaction between the two-phases as the degree of disequilibrium increases ahead of the pressure wave.

3.2- Effect of solids loading ratio:

The effect of changing the solids loading on the propagation velocity ($a_m$) is shown in Figs(6-9). The results are for different types of powder materials. Solid curves are for the case where solids volume is considered and broken curves are for the case where the solids volume is neglected. From these figures it can be seen that, the speed of sound decreases with increasing the solids loading ratio, the effect becoming more marked as the degree of velocity disequilibrium between the gas and the solids is increased. These results can be explained as being due to an increase in the solids loading ratio causing an increase in the number of particles in a given volume of gas with a consequent provide an additional pressure drop component through the mechanism of momentum exchange between the two phases and the additional turbulent energy produced in the flow. The combination of these effects causes the speed of sound to be lowered as the solids loading ratio is increased. Therefore, the maximum value of speed of sound occurs in pure air flow, where X = 0.0, and decreases with increasing, X.

On the basis of previous results reported by Mobbs, et.al. [10] , for the flow of a polystyrene-air mixture, it was anticipated that (K) would assume a value of about 0.8. In their investigation, the particle velocity was measured using streak photography technique, using a high speed cine camera. Fig. (6) shows a comparison between the experimental results and the theoretical results, for air - polystyrene mixture, calculated assuming K = 0.8. The agreement is seen to be reasonably good.

3.3- Effect of changing the solids material:

In order to investigate the effect of varying the solids material, the polystyrene powder is assumed to be replaced by one of the following materials, which usually used in the experimental investigation; glass spheres ; silica and steel shot powders.

Fig.(10) shows a theoretical results for the variation of mixture speed of sound with the velocity lag (K), for X = 10 . From this figure, it can be seen that the depression of the velocity of propagation is greater for the higher gas-solids density ratio, i.e. for the lighter particles which occupy a larger volume. These results can be explained according to the value of particle number density, which may be calculated from the following relation;

$$n_p = \frac{6}{\pi} \cdot \frac{X}{K} \cdot \frac{\rho_g}{\rho_p \cdot d_p^3}$$

Where, $n_p$ is the number of particles flowing per unit volume of flow and $d_p$ is the particle diameter, while X , K, $\rho_g$ and $\rho_p$ are defined in the nomenclature. Further-more, since the particle number density is also dependent upon the particle velocity lag, K, and other flow parameters, as shown in the above relation, any change of either, K , and , $\rho_p$ , results in a change in , $n_p$, for constant X , $\rho_g$ and $d_p$ . These effects will be more effective in the variation of speed of sound in suspension flow when the solid phase has a variable particle material density.

4. Conclusions:

The small amplitude pressure disturbances in a non-equilibrium gas - solid suspension flow was studied numerically. The velocity of propagation of a steep fronted pressure wave depends on the proportion of the total volume occupied by the solids particles, the solids loading ratio , the gas-particle density ratio (type of powder used), the direction of wave propagation relative to the flow direction and the degree of disequilibrium between the two phases.

Reasonably good agreement between theory and experiment is obtained if the behaviour of particle is predicted by considering pressure forces to predominate over viscous drag during the passage of a particle through the wave.

References:

1. Pai, S.I. and Hsieh, T., shock wave relations in Lunar Ash flow ", Tech. Note BN. 759, University of Maryland, 1973.

2. Hoglund, R.F. , " Recent Advances in gas-particle nozzle flows " , A.R.S. Journal, Vol. 32, No. 5, 1962.

3. Kliegel, J.R. , " Gas-particle Nozzle Flows" , The combustion Institute, 9th symposium on combustion, P. 811, NewYork, 1963.

4. Rudinger, G., " Some effects of particle volume on the dynamics of gas-particle mixtures ",AIAA J., Vol. 3., P. 1217, 1965.

5. Mobbs, F.R., Bowers, H.M. and Cole,B.N." A theory for the high speed flow of gas-solids mixtures under conditions of equilibrium and constant fractional lag", Inst. of Mech. Eng. and Inst.of Chem. Eng., Joint symposium on Two-phase Flow, paper No. 8., 1969.

6. Rudihger, G., " Relaxation in gas-particle Flow", Non-Equilibrium Flows, Vol.1, Part I, P.P.Wegner, Editor, Marcel Dekker, New York, P. 119, 1969.

7. Warda, H.A., " A study of gas-solids flow in supersonic nozzles ", Ph.D. Thesis, Leeds University, 1975.

8. Ibrahim, K.A., " Shock wave propagation in gas - solid suspensions", Ph.D. Thesis, Leeds University, 1980.

9. Varma, T.D., and Chopra, N.K, " Analysis of normal shock waves in gas-particle mixture", Z. Angew. Math.,Phys., Vol. 18, P. 650, 1967.

10. Mobbs, F.R. , Bowers, H.M., Riches, D.M. and Cole, B.N., " Influence of particle size distribution on the high speed flow of gas-solid suspensions in a pipe", Inst. of Mech. Eng., and Inst. of Chem. Eng., Joint symposium on Two-Phase flow, paper No.9, 1969.

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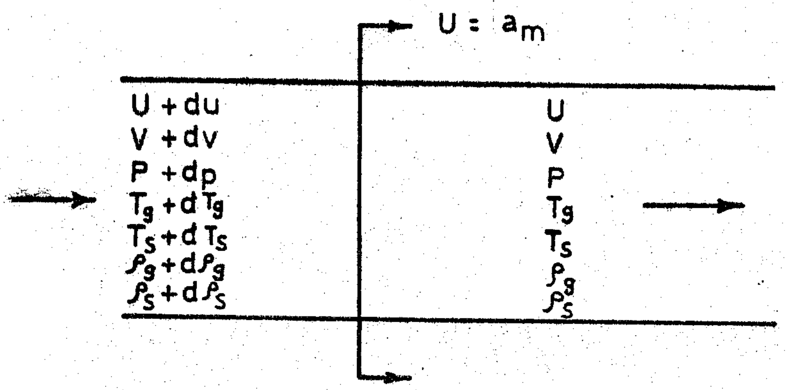



Fig.(1) Pressure wave moving downstream

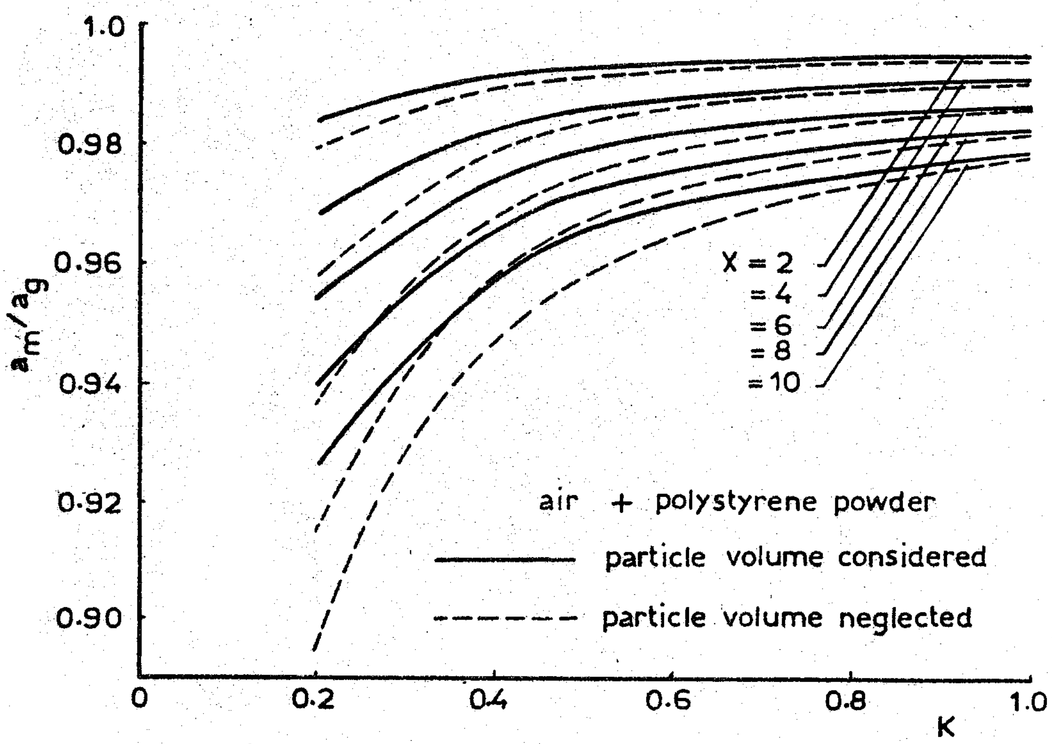



Fig.(2) Ratio between the velocity of propagation and the velocity lag (K)

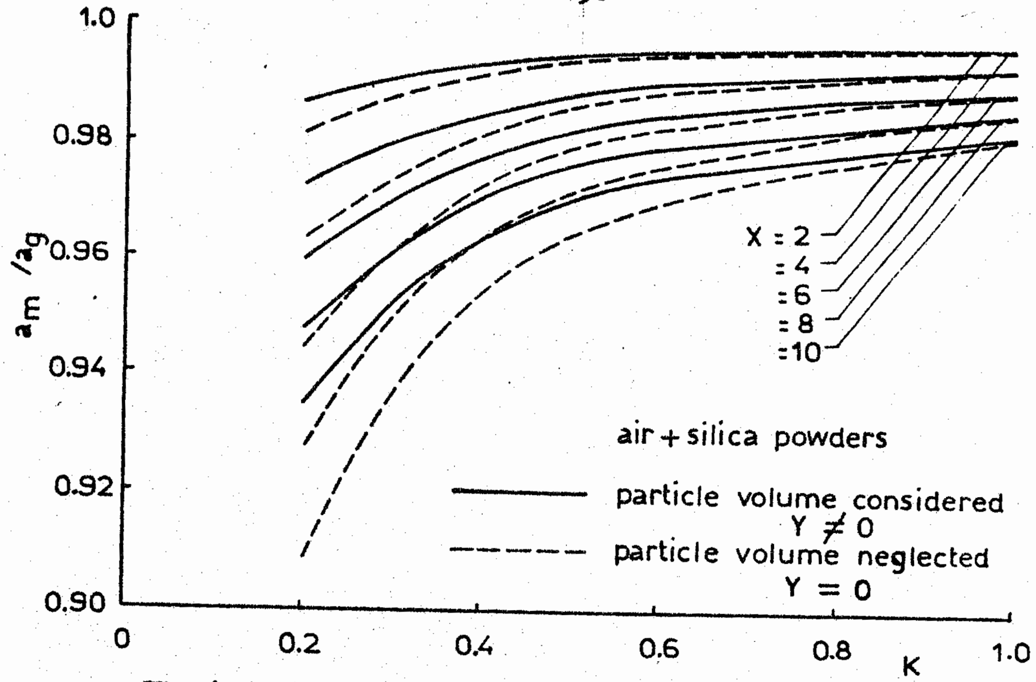
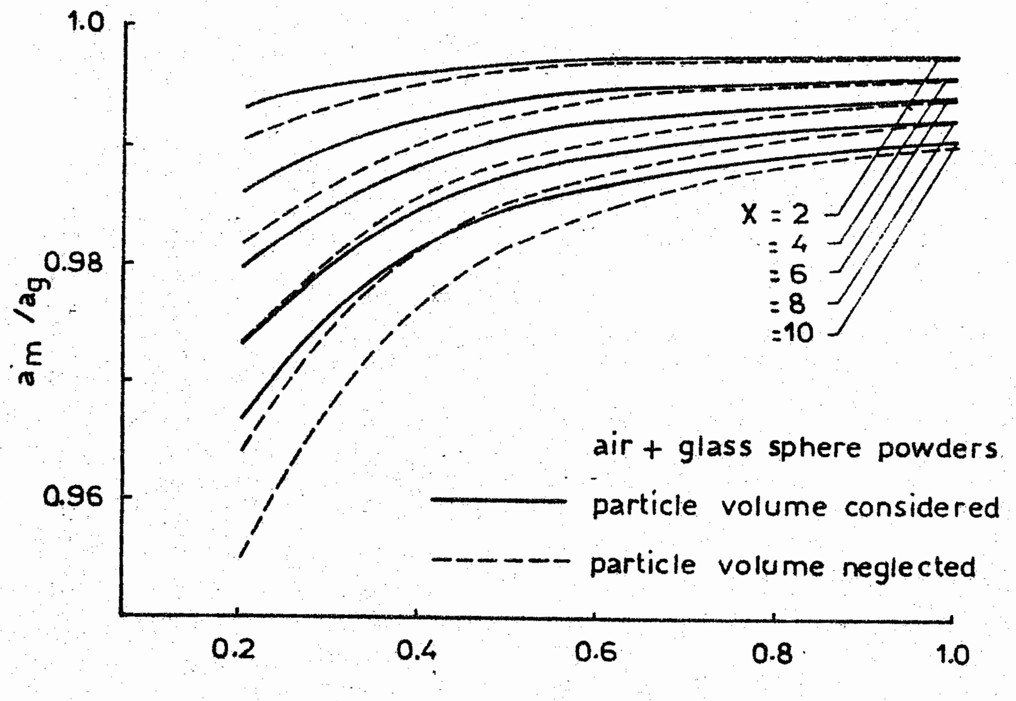

1.0
0.98
0.96
0.94
0.92
0.90
$a_m / a_g$
0
0.2
0.4
0.6
0.8
1.0
K
X = 2
= 4
= 6
= 8
= 10
air + silica powders
particle volume considered
Y ≠ 0
particle volume neglected
Y = 0

Fig.(3) CONTD.


1.0
0.98
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$a_m / a_g$
0.2
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1.0
X = 2
= 4
= 6
= 8
= 10
air + glass sphere powders
particle volume considered
particle volume neglected

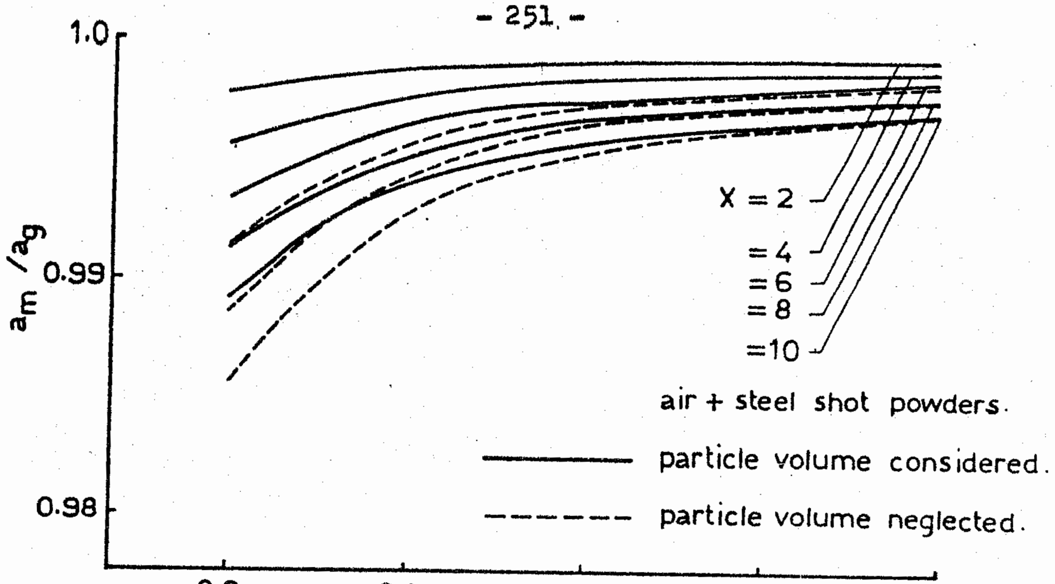


Fig.(5) CONTD.

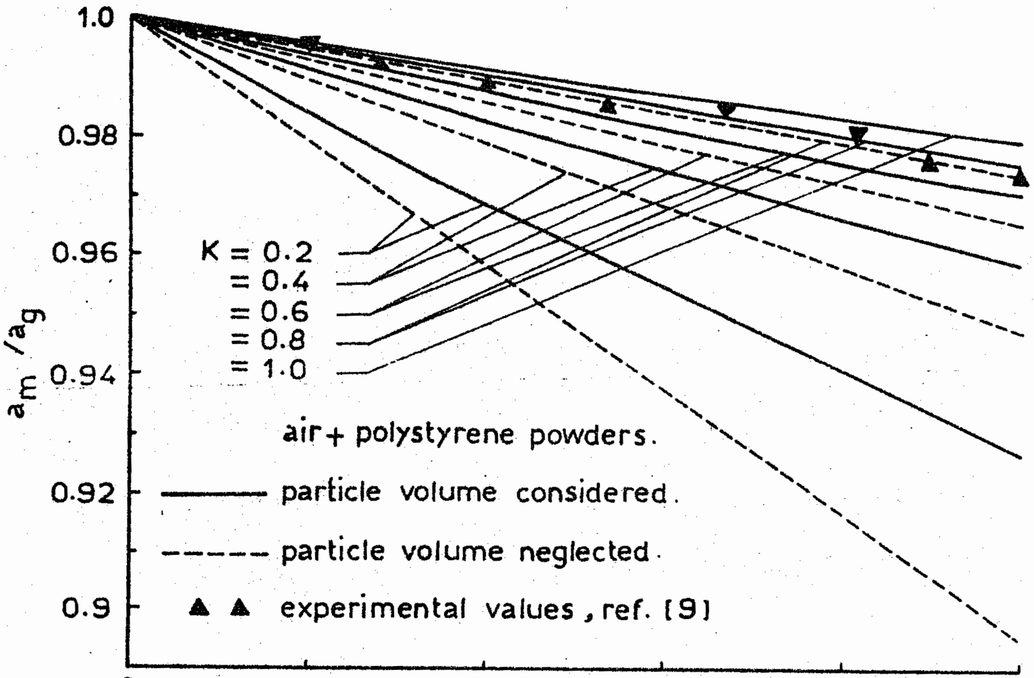


Fig.(6) Ratio between the solids loading ratio(x), and the velocity of propagation

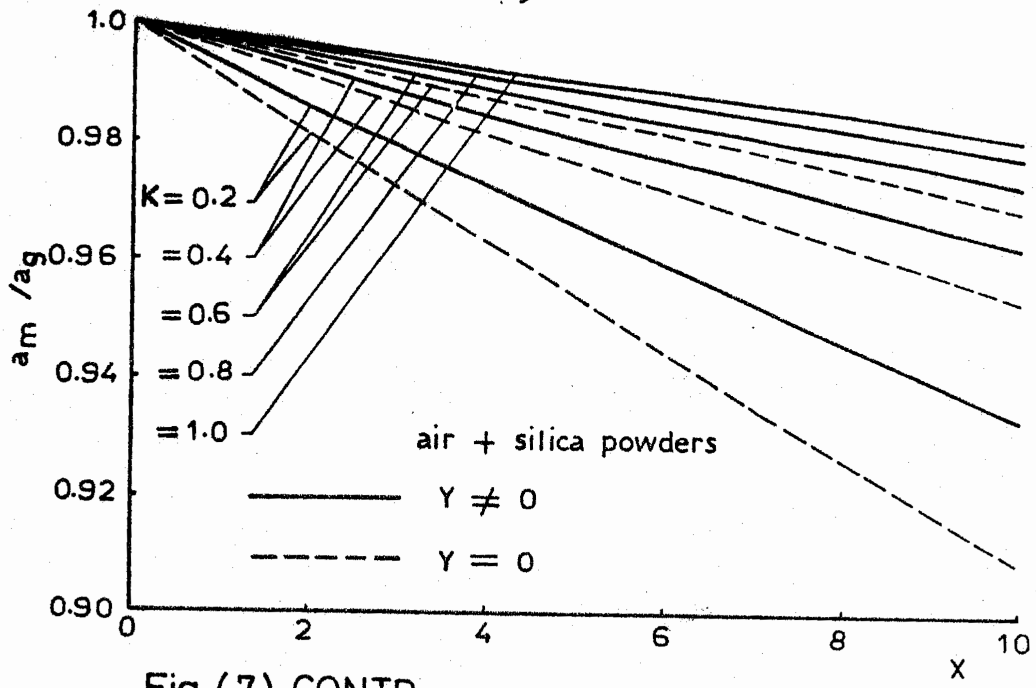


Fig.(7) CONTD.

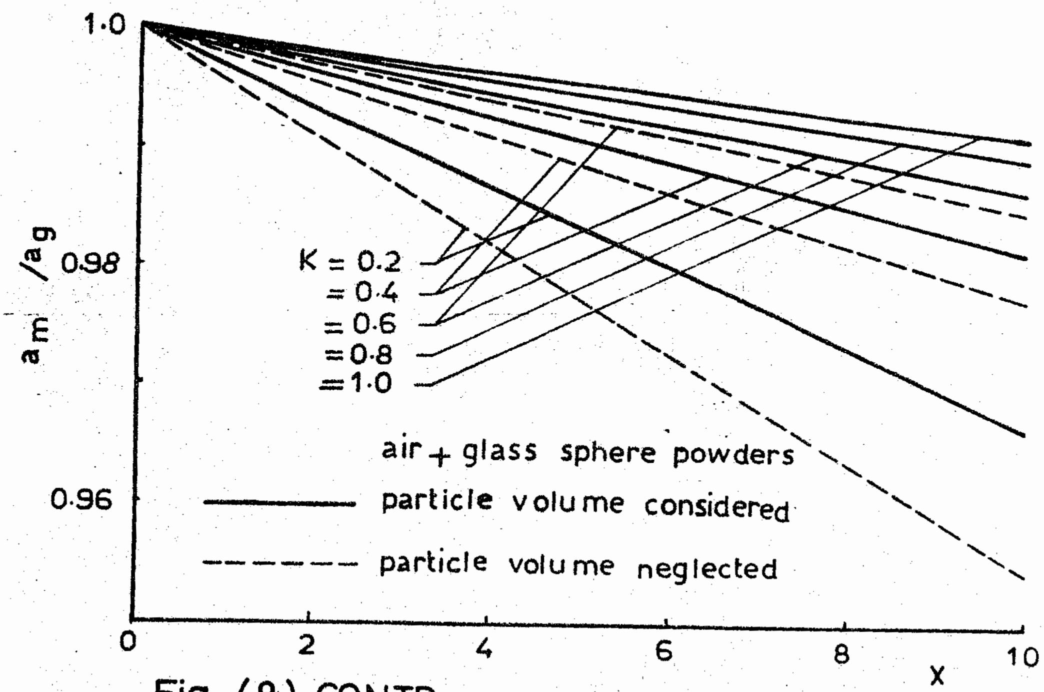


Fig. (8) CONTD.

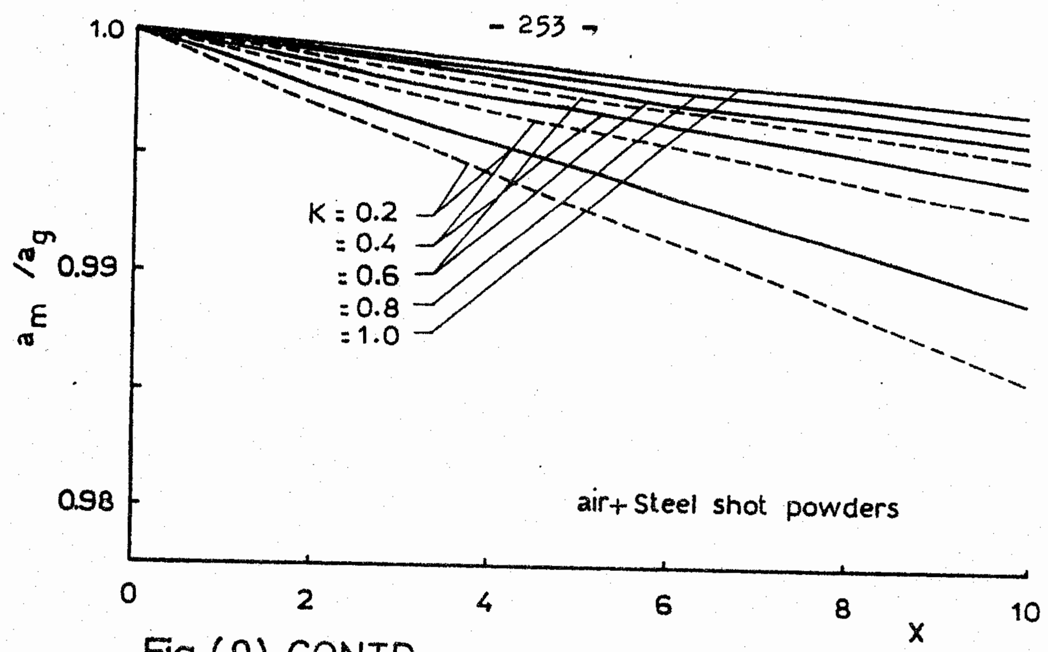


Fig.(9) CONTD.

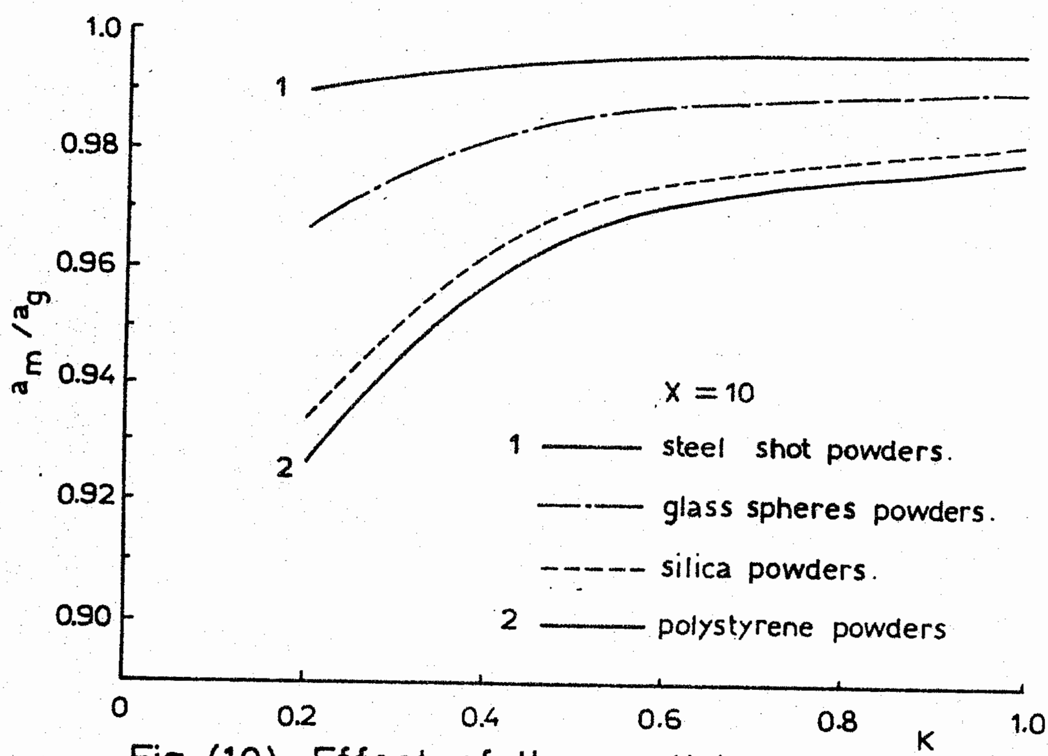


Fig. (10) Effect of the particle material density