Learning Characteristics of a new Cascaded Second Order Adaptive Filter

دراسه خصائص نوع جديد للمرشحات المتهايئه ثنائية التتابع

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ملخص

هذا البحث يقدم نوع جديد من المرشحات المتهايئه يسمي *Rcos* المتتابع. حيث تم عمل التحليل الرياضي المفصل لهذا النوع من المرشحات. و تم عمل دراسة مقارنـه بين هذا المرشح و مرشحات متهايئه أخري متعارف عليها و ذلك عن طريق المحاكـاة علي الحاسب الآلي. و قد أثبتت النتائج أن المرشح الجديد يعطي معدل تقارب أسرع مع نسبة خطأ أقل عن المرشحات الأخري. والمرشح المقترح تم تطبيقه في منظومات التوقع الخطي المتهايئ لضغط الإشارات الصوتيه.

Abstract

This paper introduces a new structure of adaptive filter called Rcos filter. Analysis of the filter is considered in details. A comparative study of performance of the filter with different structures is also considered. It is found that the adaptive Rcos cascaded second order structure outperforms the ladder, lattice, and cascaded second order structures. The new structure find its applications in linear prediction of speech signals.

1. Introduction

In the field of signal processing, it is sometimes desirable to make use of a filter which adapts itself to the input signal in such a way that the error output of the filter is minimised (e.g. to eliminate noise, interference, echos, or other unwanted signals). Such an adaptive filter is one aspect of linear prediction [1,2], in which the signal under consideration can be modeled as a linear combination of previous input and/or outputs of the filter. The traditional form is a tapped-delay-line or ladder structure digital filter[3,4, 5, 6], as shown in *Fig. 1*. However, this filter suffers from both poor convergence and sensitivity to round-off noise in practical finite word length applications. To overcome these problems a modified structure developed by cascading second order ladder sections in series was introduced [7,8]. This structure is shown in *Fig. 3*. Moreover, a lattice structure was also introduced to overcome the above mentioned problems [9,10,11]. The lattice filter structure is shown in *Fig.2*.

The Rcos cascaded second order filter coefficients are preferred in linear prediction of speech than the ladder coefficients. This is because of their closer relationship to the spectrum of the speech being coded and their uniformly distributed sensitivity to quantisation noise effects across the frequency spectrum.

2. Adaptive Ladder Filter Analysis

The basic form of adaptive linear prediction filter implemented as ladder structure [3, 4, 5, 6], is shown in *Fig.1*. It consists of a finite impulse response digital filter with variable cofficients $\{W_i\ ,\ 1 \le i \le M\}$. A signal vector $\underline{X}$ consisting of the output signals from the delay line elements can be defined as :

* Accepted February 14, 2000

$$\underline{X}^T = [x(n) \quad x(n-1) \quad \ldots\ldots\ldots\ldots\ldots \quad x(n-M)] \tag{1}$$

A prediction signal $\hat{x}(n)$ of the input signal $x(n)$ at time n based on the M previous samples $x(n-1), x(n-2), \ldots\ldots\ldots\ldots\ldots\ldots\ldots x(n-M)$ can be given as:

$$\hat{x}(n) = \sum_{i=1}^{M} w_i(n) \cdot x(n-i) \tag{2}$$

where $\{w_i(n)\}$ are the prediction coefficients at time n, in matrix notation form, equation (2) can be written as:

$$\hat{x}(n) = \underline{W}^T \cdot \underline{X} \tag{3}$$

Where

$$\underline{W}^T = [w_1(n) \quad w_2(n) \quad \ldots\ldots\ldots\ldots\ldots \quad w_M(n)]$$

The prediction error signal $e(n)$ at time n is defined as:

$$e(n) = x(n) - \hat{x}(n)$$

$$= x(n) - \underline{W}^T \cdot \underline{X} \tag{4}$$

Minimisation of the mean square error may be carried out by the "Least-mean-square" algorithm (LMS)[12], which requires that the coefficients $\{w_i, 1 \leq i \leq M\}$ to be updated at each sampling instant n, as,

$$\underline{W}(n+1) = \underline{W}(n) + 2\mu\, e(n)\, \underline{X} \tag{5}$$

The constant μ is a step -size controls the stability and the rate of convergence.

3. Adaptive Lattice Filter Analysis

Lattice filters [10,11,12] of the type, shown in *Fig.2* ,have coefficients K's less than 1 in modules, and the forward and backward errors are all minimized in mean square value when the output is similarly minimised (orthogonality property) .

The forward and backward errors at each stage of the lattice filter are given by :

$$f_i(n) = f_{i-1}(n) + k_i\, b_{i-1}(n-1) \tag{6}$$

$$b_i(n) = b_{i-1}(n) + k_i\, f_{i-1}(n) \tag{7}$$

with initial values given by :

$$f_0(n) = b_0(n) = x(n) \tag{8}$$

The forward error generated by the final stage of the lattice, can be expressed in the form :

$$e(n) = f_M(n)$$

$$= f_0(n) + \sum_{i=1}^{M} k_i\, b_{i-1}(n-1) \tag{9}$$

Equation (9) can be expressed in terms of the forward and backward errors at any i of the lattice filter as follows

$$e(n) = f_{i-1}(n) + k_i \cdot b_{i-1}(n-1) + \sum_{j=i+1}^{M} k_j b_{j-1}(n-1) \qquad 1 \leq i \leq M \tag{10}$$

Minimisation on the mean square error may be carried out by the "Least-mean-square" (LMS) algorithm, which requires that the coefficients $\{k_i, 1 \leq i \leq M\}$ be updated at each sampling instant n, as,

$$k_i(n+1) = k_i(n) - \mu \frac{\partial e^2(n)}{\partial k_i(n)}$$

$$k_i(n+1) = k_i(n) - 2\mu \, e(n) \frac{\partial e(n)}{\partial k_i(n)} \tag{11}$$

The constant μ is a step -size controls the stability and the rate of convergence as before. Differentiating equation (10) gives

$$\frac{\partial e(n)}{\partial k_j(n)} = b_{j-1}(n-1) + \sum_{i=j+1}^{M} k_i(n) \frac{\partial b_{i-1}(n-1)}{\partial k_j(n)} \tag{12}$$

The change of k_j should in theory have immediate effect, not only on the output error $e(n)$, but also on the stored backward errors $\{ b_{i-1}(n-1),\ 1 \le i \le M \}$ hence the $\frac{\partial b_{i-1}(n-1)}{\partial k_j(n)}$ terms should, in theory, be non-zero. Since it is impractical to recalculate the backward errors as adaptation proceeds, they in fact remain, in short terms, independent of changes in the filter coefficients, therefore we can use the approximation.

$$\frac{\partial e(n)}{\partial k_j(n)} \cong b_{j-1}(n-1) \qquad ; \ j=1,2, \dots\dots\dots,M \tag{13}$$

therefore

$$\frac{\partial e^2(n)}{\partial k_i(n)} \cong 2e(n)\ \underline{b}(n)$$

where $\underline{b}(n) = [b_0(n-1) \quad b_1(n-1) \quad \dots\dots\dots\dots \quad b_{M-1}(n-1)]^T$.

the simplified " end-point" updating algorithm becomes

$$k_i(n+1) = k_i(n) - 2\mu \, e(n) \, b_{i-1}(n-1) \quad ; i=1,2,\dots M \tag{14}$$

4. Adaptive Cascaded Second Order Filter Analysis

The adaptive filter may also be constructed by cascading second order ladder sections in ,series [8,9], as shown in *Fig. 3*

The filter is M^{th} order with $L = \frac{M}{2}$ sections, if M is even. An odd order filter (*i.e* M is odd) can also be obtained by cascading $L = \frac{M+1}{2}$ sections in cascade. The output $e_i(n)$ of an intermediate section of the filter is given by :

$$e_i(n) = e_{i-1}(n) + a_{i1}\, e_{i-1}(n-1) + a_{i2}\, e_{i-1}(n-2) \ ; \ 1 \le i \le M \tag{15}$$

with

$$e_0(n) = x(n)$$

and

$$e(n) = e_L(n) \tag{16}$$

The filter parameters a_{ij} ; $i = 1,2, \dots\dots\dots\dots L$ and $j = 1,2;$ are updated sequentially by the LMS algorithm as follows

$$a_{ij}(n+1) = a_{ij}(n) - 2\mu \, e(n)\, g_{ij}(n) \tag{17}$$

where $g_{ij}(n)$ are the gradient components calculated as

$$g_{ij}(n) = \frac{\partial\, e(n)}{\partial\, a_{ij}}$$

$$= e(n-j) - a_{i1}\, g_{ij}(n-1) - a_{i2}\, g_{ij}(n-2) \qquad (18)$$

5. Adaptive Rcos Cascaded Second Order Filter Analysis

In this section, a new cascaded second order structure is introduced. This structure is called *Rcos* adaptive filter, and is composed of cascaded second order filter structure as shown in *Fig. 4.* To understand the idea of the new structure let us perform the following analysis.

The transfer function of the cascaded second order filter can be obtained by taking the Z transform of the error equations of the cascaded second order filter in equation *(15)* and *(16)* as follows

$$E_i(z) = (1 + a_{i1}\, Z^{-1} + a_{i2}\, Z^{-2})\, E_{i-1}(z) \qquad (19)$$

with

$$E_0(z) = X(z)$$

and (20)

$$E(z) = E_L(z)$$

$$\therefore\ E(z) = \prod_{i=1}^{L}(1 + a_{i1}\, Z^{-1} + a_{i2}\, Z^{-2})\, X(z) \qquad (21)$$

and the transfer function $A(z) = \frac{E(z)}{X(z)}$ is given by

$$A(z) = \prod_{i=1}^{L} A_i(z) = \prod_{i=1}^{L}(1 + a_{i1}\, Z^{-1} + a_{i2}\, Z^{-2}) \qquad (22)$$

where $A_i\,(z)$ is the transfer function of the i^{th} section.

In order to update the filter parameters using the *LMS* algorithm, the gradient of the error $g_{ij}\,(n)$ is given as :

$$g_{il}(n) = \frac{\partial\, e(n)}{\partial\, a_{il}} \qquad (23)$$

using *Z*- transform, equation *(23)* is expressed as :

$$G_{il}(z) = Z^{-1} \prod_{i=1}^{L}(1 + a_{i1}\, Z^{-1} + a_{i2}\, Z^{-2})\, X(z) \qquad (24)$$

where

$$i = 1,2, \ldots\ldots\ldots\ldots L\ \ \&\ \ j = 1,2$$

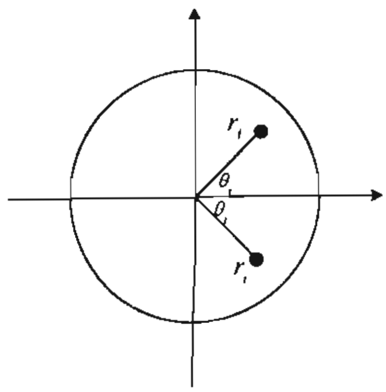
Comparing *(19)* and *(24)*, then

$$G_{il}(z) = \frac{Z^{-1}\, E(z)}{1 + a_{i1}\, Z^{-1} + a_{i2}\, Z^{-2}} = Z^{-1}\, A_i^{-1}(z)\, E(z) \qquad (25)$$

and $A_i^{-1}(z)$ corresponds to the inverse transfer function. The filter coefficients are updated as

$$a_{il}(n+1) = a_{il}(n) - 2\mu\, e(n)\, g_{il}(n) \qquad ;0 < \mu < \frac{1}{\lambda_{max}} \qquad (26)$$

If the roots of each section of cascaded se ctions occur in a complex conjugate pairs with magnitude r_i and phase θ_i where:



$$r_i = \sqrt{a^2 + b^2}$$

and

$$\theta_i = cos^{-1} \frac{b_i}{a_i}$$

Then $A^{-1}(z)$ may be expressed as

$$\begin{aligned} A^{-1}(z) &= [z-(a+jb)]\,[z-(a-jb)] \\ &= z^2 - (a+jb)\,z - (a-jb)\,z + (a^2+b^2) \\ &= z^2 - 2a\,z + (a^2+b^2) \\ &= z^2 - 2r\cos\theta\,z + r^2 \\ &= z^2\left[1 - 2r\cos\theta\,z^{-1} + r^2\,z^{-2}\right] \end{aligned}$$

Therefore, from the above analysis we get,

$$a_{i1} = -2r_i cos\theta_i$$

and (27)

$$a_{i2} = r_i$$

Then the error of the i^{th} section of the rcos cascaded second order filter is given as :

$$e_i(n) = e_{i-1}(n) + a_{i1}\,a_{i2}\,e_{i-1}(n-1) + a_{i2}^2\,e_{i-1}(n-2) \qquad ;1\leq i\leq L \qquad (28)$$

It can easily be shown that the filter parameter a_{il} where $i = 1,2,\ldots\ldots\ldots, L$ and $l=1,2$ are related to the zero positions r_i and θ_i by

$$a_{i1} = -2cos\theta_i$$

and (29)

$$a_{i2} = r_i \qquad 1\leq i\leq L$$

In the synthesis filter, the zeros of the prediction filter become the poles of the synthesis filter. A pole of the transfer function shows up as a peak in the spectrum unless its bandwidth is too wide or it is masked by some other feature of the transfer function. Thus, the location of the pole $r_i e^{\pm j\theta_i}$ is closely related to the center frequencies F_i and bandwidth B_i of the peak in the power spectrum which are the speech formants given by

$$F_i = \frac{\theta_i}{2\Pi T}$$
$$B_i = \frac{|ln\ r_i|}{\Pi T} \qquad (30)$$

and T is the sampling period.

Therefore, when applying the *LMS* algorithm to adapt the $Rcos\theta$ cascaded second order filter parameters a_{il} we essentially vary the radii and angels of the transfer function poles to give a better estimate of the spectral envelope of the speech segment under consideration.

The constraints that are to be placed on the coefficients to ensure complex conjugate zeros of the prediction filter within the unit circle of Z-plane are :

$$|a_{i1}| < 2$$

and $$0 < a_{i2} < 1$$

6. Experimental Results

The behavior of the different structures of adaptive digital filters using *LMS* adaptive algorithm, has been studied for stationary input signals. *Fig.5* shows a block diagram for the computer simulation model.

One very useful way to monitor the progress of the adaptive process is to plot its "learning curve"[13]. Since the basic idea of adaptive prediction is to adjust the adaptive filter parameters so as to minimize the mean square error of the output value as discussed before, it seems logical to use the mean squared error as a criterion. The expected mean squared error at each stage of the learning process is thus plotted as a function of the number of adaptive iterations.

In order to compare and assess the results shown in *Fig.6*, the misadjustment (*Ma*) criteria, as defined by widrow[14,15] is used. Since the adaptive filters will not perfectly adapt to the optimum because of random fluctuations due to gradient estimation noise, even after equilibrium has been approached, the result is a mean square output error greater than the optimum. The amount by which it is greater is called "Excess Mean Square Error" (*EMSE*) as indicated in *Fig.6*. The measure of the extent to which the adaptive filter is misadjusted as compared to the optimum is determined by the ratio of the excess mean square error to the optimum,

$$MA = \frac{Average\ \ Excess\ \ Mean\ \ Square'\ Error}{Optimum\ \ Mean\ \ Square\ \ Error}\ \%$$

A series of experiments was carried out by computer simulation to investigate the convergence behavior of the different filter structures, for stationary input signals $x(n)$, as shown in *Fig.5*, a "white noise" generated by exciting a fixed all-pole recursive filter with Gaussian noise of zero mean and unit variance. The position of the poles could be varied by changing the coefficient of the fixed filter.

The output from the all-poles fixed filter was passed through the adaptive filter whose coefficients were set initially to zero. The adaptive filter was allowed to update its coefficients on a sample-by-sample basis for period of up to 600 samples. The final 400 samples in each experiment is used to find the misadjstment.

Table (4-1) shows values of misadjustment (MA) using different values of the step-size μ. *Fig. (7)* shows the misadjustment variation with respect to the step size μ for the four structure of adaptive filters.

Table(4-1) *Values of Misadjustment (MA) for Different Values of the Step-Size* μ

μ	Ladder Structure	Lattice Structure	Cascade 2nd order Structure	Rcos Cascading Structure
0.0005	33.5	2.2	3.1	0.7
0.001	50.2	2.9	3.9	1.1
0.0025	71	3.4	4.9	1.7
0.005	106.7	5.1	6.5	2.4
0.0075	210	6.2	35	3.1
0.01	Over flow	36	103	3.9
0.015	Over flow	209	over flow	55

7. Conclusion

This paper developed a new kind of second order cascading filter known as rcos cascading filter and demonstrated a performance comparison between this filter and ladder, lattice and cascade second order filter . It has been verified by computer simulation experiment that :

1- The misadjustment increases as a step size μ increases in all structures.

2- The rcos cascading second order filter provides better improvements in performance followed by the cascade second order filter followed by the lattice order followed by the ladder filter.

3- The rcos cascading structure demonstrated superior misadjustment followed by lattice filter followed by the cascade second order filter followed by the ladder filter.

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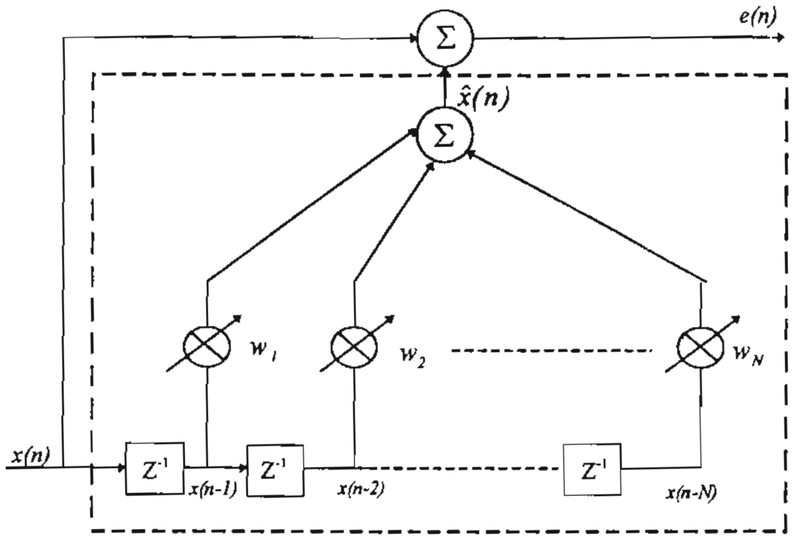

Fig.1 An Adaptive Predictive Filter in Ladder Form

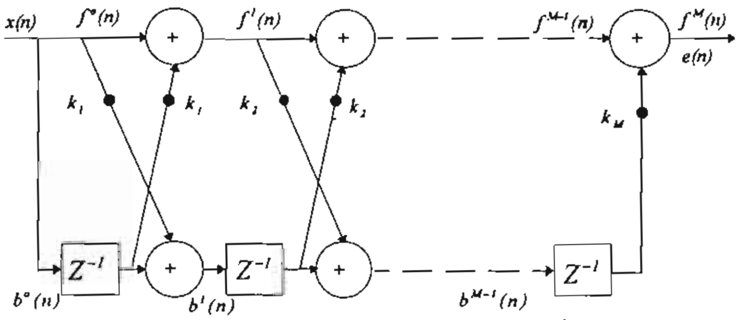

Fig.2 Lattice Structure Prediction Filter of Order M

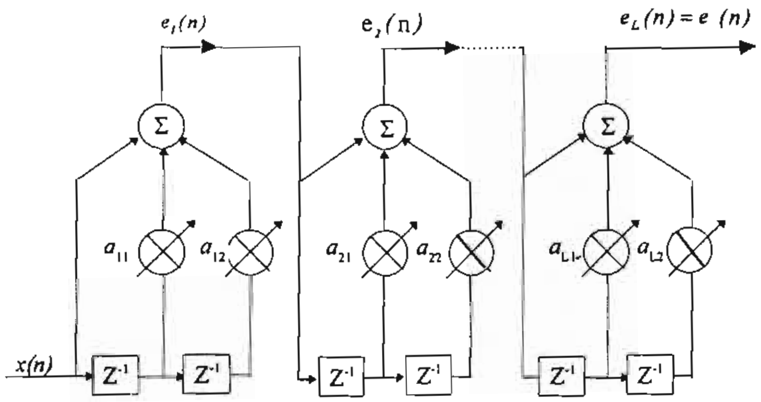

Fig.3 An Adaptive Cascaded Second Order Predictive Filter

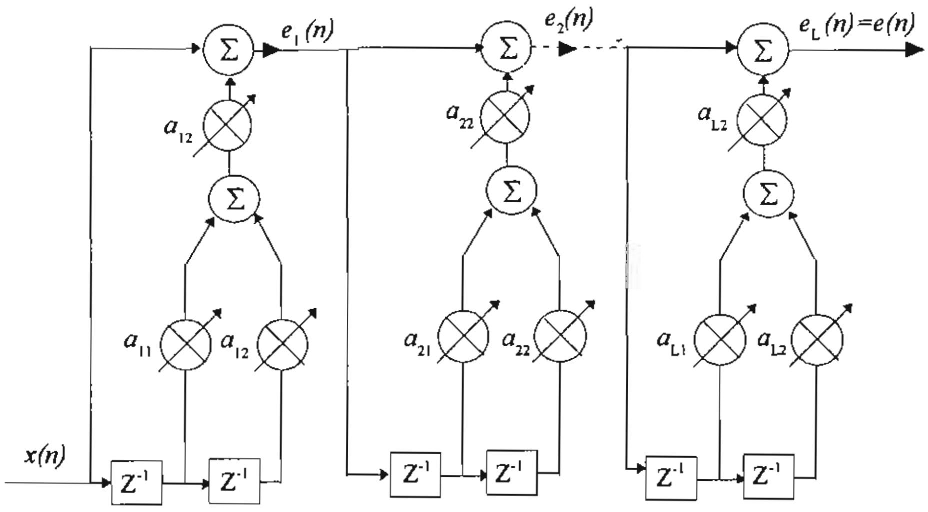

Fig.4 An Adaptive Rcos Cascaded Second Order Predictive Filter

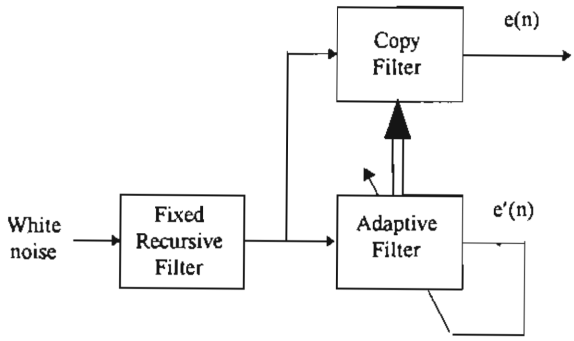

Fig.5 Computer Simulation Model

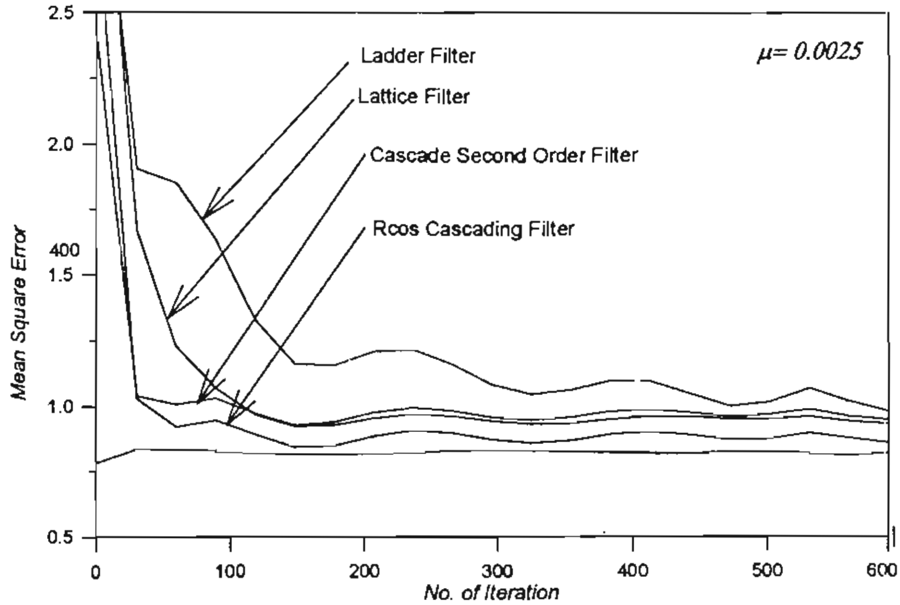

***Fig.6** Ensamble Average Learning Curves of Four Structures of Adaptive Filters*

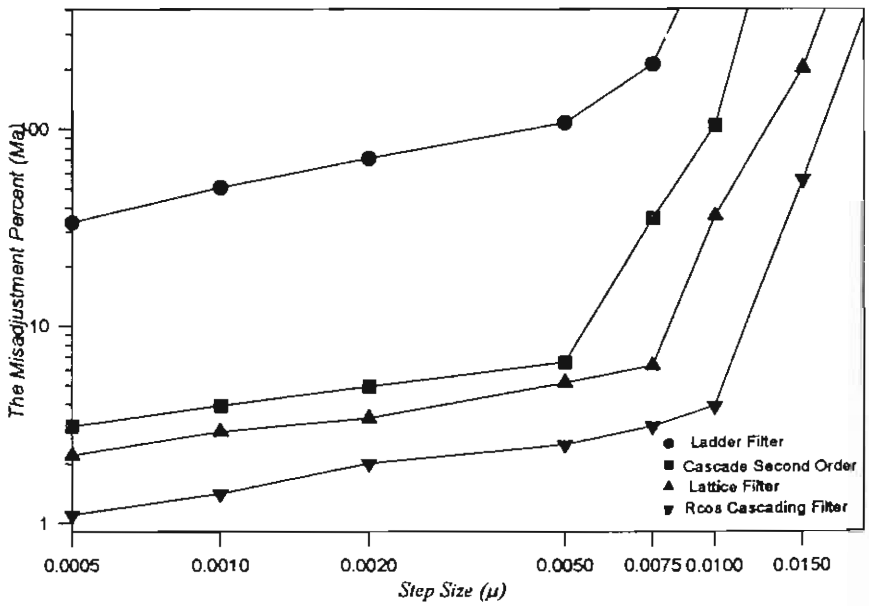

***Fig. 7** The Misadjustment Variation as a Function of step Size μ*