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LAMINAR FILM CONDENSATION OUTSIDE AN INCLINED TUBE

تكثيف الفيلم الرفائقى على السطح الخارجى لأ نبوبة مائلة

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ملخص :

انتقال الحرارة فى الفيلم الرفائقى لبخار مشبع نقى ساكن يتكاثف على السطح الخارجى لأنبوبة مائلة تم تحليله بتطبيق مبادىء الطبقة الحدية الرقائقية. ونتج عن هذا التحليل معادلة تفاضلية جزئية لحساب سمك طبقة التكثيف، والتى تعطى فى الحالة الرأسية للأنبوبة حل ناسلت (Eq. 19c) بينما فى الحالة الرأسية للأنبوبة فإنها تؤل إلى معادلة تفاضلية عادية والتى تؤكد (وتثبت) أن سمك الفيلم عند قمة الأنبوبة لايساوى صفر كما فرض ناسلت فى تحليله. وباعتبار سمك الفيلم الجديد (وليس صفراً كما فرض ناسلت) فى الحل الحالى أمكن الحصول على معادلة (Eq.23) هى تطوير لمعادلة ناسلت والتى فيها الثابت يساوى ٠,٦٥ بدل من ٠,٧٢٨ فى معادلة ناسلت الأصلية لحساب معامل انتقال الحرارة المتوسط و يمكن اعتبار هذا التغيير والذى نتج من التحليل الحالى فى حل ناسلت الأصلى فى الأتجاه الصحيح ، حيث أن بعض الباحثين قد أوصوا بتغيير هذا المعامل الثابت من ٠,٧٢٨ الى ٠,٦١ فى معادلة ناسلت للأنبوبة الأفقية وذلك لتحقيق توافق جيد مع القياسات المعملية. بالنسبة للأنبوبة المائلة تم حل المعادلة التفاضلية الجزئية (الرئيسية Eq. 17) حلاً عددياً لحساب سمك الفيلم ، وقد تم تحديد قيم سمك الفيلم على طول الأنبوبة (φ =0) تحليلياً (أنظر Eq.28) لتستخدم كظروف حديه فى الحل العددى وذلك حتى يتحقق حلاً بدون فروض حدية تقريبية. هذا وتم الحصول على نتائج عددية لزوايا ميل للأنبوبة (مع الأفقى) فى المدى من (٠°-٩٠°). وقد أظهرت النتائج العددية أن زاوية ميل أقل من ١٥ ° مع الأفقي ينتج عنها زيادة فى معامل انتقال الحرارة بالنسبة للحالة الأفقية ، وبالنسبة للأنبوبة الرأسية فأن ميل بسيط (مثلا ٥ °) مع الرأسى ينتج عنها زيادة عالية جداً فى معامل انتقال الحرارة.

ABSTRACT

The heat transfer in laminar film condensation of pure saturated stagnant vapour outside an inclined tube is analyzed by applying the principles of the laminar-boundary-layer theory. The analysis results in a partial differential equation for the local film thickness, which in the vertical case, yields the result found by Nusselt. However, for the horizontal case, this equation reduces to an ordinary differential equation which verifies that the film thickness at the top point of a horizontal tube is not zero as assumed by Nusselt, and certain value has been determined. This value is of order of the critical thickness of the stable film; defined by another author studied the characteristic of transient, and steady condensate film. Consequently, a modified Nusselt formula has been determined for calculating the average heat transfer coefficient, in which the constant coefficient is 0.65 instead of 0. 728 in the Nusselt solution. This modification may be in the right direction in view of some authors proposed such a modification for better agreement with measured data. Then, the main partial differential equation has numerically been solved for the inclined tube, and results obtained for inclination angles with the horizontal; from 0° to 90°. The exact solution of the vertical, and that of the horizontal tube are used to test the numerical solution. The results indicate that small inclination angle with the horizontal ($\leq 15^\circ$) results in an increase in the heat transfer coefficient over the horizontal value. While for a vertical tube, small inclinations from the vertical position, ($\leq 5^\circ$) results in a significant increase in the heat transfer coefficient above the vertical value. Comparison of present results with available correlations has been made.

1. INTRODUCTION

Condensation of vapour is an important phenomenon which occurs in numerous engineering applications. The heat transfer in condenser pipes is normally dominated by the heat transfer resistance of the liquid film, therefore, the film thickness is one of the most important properties to predict the local heat heat flux. Nusselt [1] found formula for calculating the average heat transfer coefficient of laminar film condensation of pure saturated stagnant vapour on a horizontal tube surface, which is:

$$Nu = 0.728\ (R_a\ /Ja)^{1/4} \tag{1.1}$$

where Nusselt number, $Nu = h_{av} D/k$, Jacob number, $Ja = C_p (T_s - T_v)/\lambda_{fg}$, Rayleigh number, $R_a = g(\rho-\rho_v) D^3 Pr/\rho\nu^2$; and Prandtl number, $Pr = C_p\mu/k$. (properties notions without subscripts are of liquid film). Since then, studies have been made on the laminar film, and various modifications for Nusselt solution have been proposed. Koh et al. [4] carried out a similarity solution for the complete boundary layer equations of laminar film of a stagnant saturated vapour. They found that interfacial shear and convection in the liquid film have a negligible effect compared to Nusselt's results except for low Prandtl number fluids. Rohsenow [5] proposed a corrected latent heat: $\lambda^*_{fg} = \lambda_{fg} + .68 C_p (T_s - T_v)$ to account for subcooling in film.

Because of the small vertical dimension of a horizontal tube; the film path length is short as compared with a vertical surface case, therefore, it is rarely turbulence in the film [11]. Selin [6] condensed several alcohols on three different-diameters horizontal copper tubes. He found that better predictions by Nusselt solution can be made if the constant value of 0.728 is changed to 0.61, while he also recommended a modification in the opposite direction in Nusselt solution of a vertical plate, as previously proposed by McAdams [7]. In recent studies, Nusselt solution of a vertical surface has been extended to account for the effects of vapour velocity and film flow turbulence [8].

Wilson [9] applied the method of characteristic to treat the problem of transient film condensation of stagnant saturated vapour on horizontal cylinders. He found that the film becomes stable when its thickness atop a horizontal cylinder reaches a certain value estimated by [3] :

$$\delta_{\varphi=o} = 1.11\ D\ (R_a\ /Ja)^{-1/4} \tag{1.2}$$

This weakens Nusselt assumption of zero film thickness at the top point of the horizontal tube.

Hassan and Jakob [10] analytically treated the film condensation on a single inclined tube applying the Nusselt assumption. They found that the average heat transfer coefficient could be correlated by Nusselt value (Eq. 1.1) multiplied by $(\cos\alpha)^{.25}$, where α is the inclination angle with the horizontal. With this modification Selin [6] could correlated data within ± 15% for $0^\circ \le \alpha \le 60^\circ$. Kröger [12] investigated vapour condensation inside an inclined tube, and observed that the heat transfer coefficient reaches a maximum value at inclination angles ranged from 10-20 deg.

In the present work, a physical model is developed for laminar film of pure saturated stagnant vapour condensing outside an inclined tube. Analytical solution will be obtained for the vertical, and for the horizontal tube. Then, a numerical scheme is developed for the solution of the inclined tube.

2. Physical Model

The present analysis considers an inclined tube with an outer diameter D, and a length L, in an infinite medium of stagnant pure saturated vapour. The outer surface temperature T_w, is assumed uniform and sufficiently small ($T_w < T_s$) to create a condensate film, which will be driven by gravity in the axial direction as well in the tangential direction along the tube periphery Provided that the film thickness is small compared to the tube diameter so that the curvature effect is neglected, and then the condensate film may be analyzed using cartesian instead of cylindrical coordinates. A schematic diagram of the physical model and coordinate system is illustrated in Fig. 1.

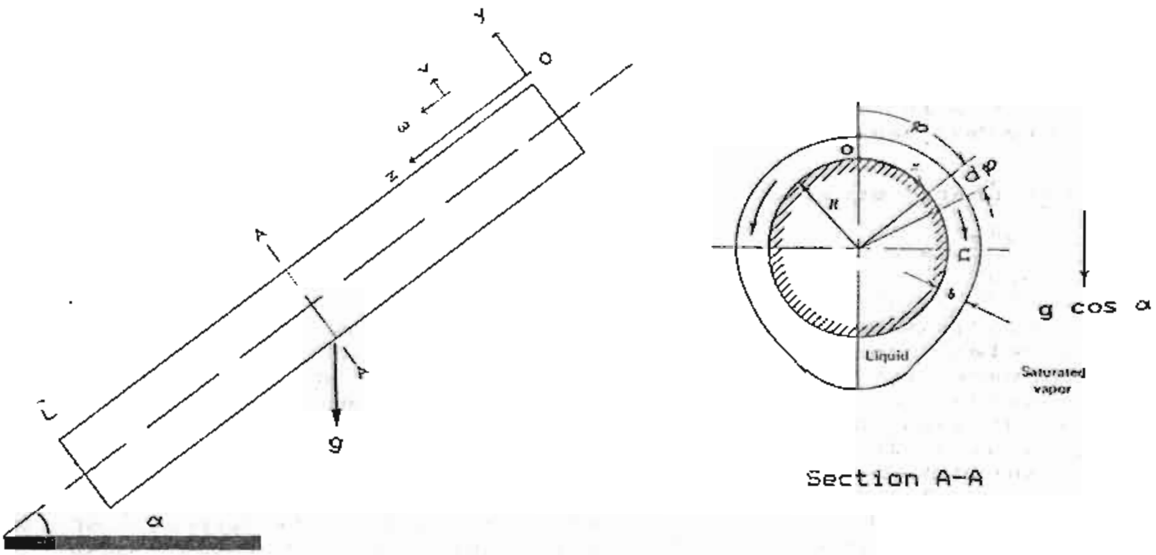

Fig.1 The physical model and coordinate system.

Assuming steady laminar condensate film flow with smooth liquid-vapour interface, pure vapour, negligible dissipation, and constant physical properties, and ignoring the effects of change in momentum and energy of the film, the principles of laminar, single-phase boundary layer theory may be applied to analyze the condensate film. It is further assumed that the static pressure in both the vapour and the liquid film is identical. Justification for ignoring various terms in the conservation equations of the film due to the above assumptions, as well as discussion of their proper approximate modeling; if it is found necessary and appropriate to be introduced in the solution, will be given through the analysis. In the text, all notions without subscript belongs to liquid film.

In this situation, the simplified governing equations for the laminar condensate layer may be given as follows :

(a) Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial z} = 0 \qquad (1)$$

here u, v and ω are the velocity components in x-, y- and z-directions, respectively.

(b) x-Component of momentum equation

$$\mu \frac{\partial^2 u}{\partial y^2} + g(\rho - \rho_v) \cos\alpha \sin\psi = 0 \qquad (2)$$

wherein the x-component of the pressure gradient in the liquid film has been substituted by: $\partial p/\partial x = g\rho_v \cos\alpha \sin\psi$, which is the same static pressure gradient of the vapour. Also, the x-component of gravity acting tangentially to the tube surface is $g\rho \cos\alpha \sin\psi$.

(c) z-Component of momentum equation

$$\mu \frac{\partial^2 \omega}{\partial y^2} + g(\rho - \rho_v) \sin\alpha = 0 \qquad (3)$$

in which the z-component of the pressure gradient in the liquid film has been inserted by : $\partial p/\partial z = g\rho_v \sin\alpha$.

(d) Energy equation

$$\frac{d^2 T}{d y^2} = 0 \qquad (4)$$

It is noted that the above equations do not include neither the acceleration terms in the momentum equations (2) and (3) nor the convection terms in the energy equation (4). Koh et al. [4] obtained a similarity solution for the complete boundary layer equations including these terms. Their results showed that the inclusion of the acceleration terms has little effect on the condensation heat transfer for Prandtl number greater than 0.1; i.e., all Prandtl numbers above the liquid-metal range. However, for smaller Prandtl number ($Pr \ll 0.1$), the effect of the acceleration terms can be significant. They also found that the executed temperature distributions within the condensate film are almost linear for values of Jacob number, $Ja = C_p(T_s - T_v)/\lambda_{fg}$, near 1. Since $Ja \ll .1$ for most of condensation applications with fluids such as water, refrigerants and hydrocarbons, therefore, neglecting the effects of convection terms in the energy equation seems to be also reasonable approximation.

Now, to complete the model formulation, the following boundary conditions are introduced

$$\text{at} \quad y = 0 \qquad u = v = \omega = 0, \qquad T = T_v \qquad (5)$$

$$\text{at} \quad y = \delta \qquad \frac{\partial u}{\partial y} = \frac{\partial \omega}{\partial y} = 0, \qquad T = T_s \qquad (6)$$

The heat balance at the liquid-vapour interface yields

$$m''_c \lambda_{fg} = -k \left[\frac{\partial T}{\partial y}\right]_{y=\delta}$$

where λ_{fg} is the latent heat of condensation, and $\ddot{m}_c$ is the mass flux of condensation through the liquid-vapour interface, which is calculated from the mass balance at the liquid-vapour interface by

$$\ddot{m}_c = \rho\, v_{y=\delta} \tag{8}$$

Integrating of the continuity equation (1) across the liquid film under boundary condition (5), yields

$$v_{y=\delta} = -\frac{\partial}{\partial x}\int_0^{\delta} u\,dy - \frac{\partial}{\partial z}\int_0^{\delta} \omega\,dy \tag{10}$$

Integration of Eq. (4) with boundary conditions (5) and (6) gives the linear temperature distribution :

$$T = T_v + (T_s - T_v)(y/\delta) \tag{11}$$

Combining equations (7) to (11) gives the following relation

$$\frac{\partial}{\partial x}\int_0^{\delta} u\,dy + \frac{\partial}{\partial z}\int_0^{\delta} \omega\,dy = \frac{k\Delta T}{\rho\lambda_{fg}\delta} \tag{12}$$

Integration of Eq. (2) across the film thickness with boundary conditions (5) and (6) yields the x-component velocity profile as

$$u = \frac{g(\rho-\rho_v)\cos\alpha\sin\psi}{\mu}\,\delta^2\left[\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right] \tag{13}$$

Similarly, integration of Eq. (3) with respect to y, toget r ith boundary conditions (5) and (6) gives the z-component velocity profile by

$$\omega = \frac{g(\rho-\rho_v)\sin\alpha}{\mu}\,\delta^2\left[\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right] \tag{14}$$

By inserting u and ω from Eqs. (13) and (14), respectively, into Eq. (12), one obtains the local film thickness equation as

$$\frac{D}{4}\sin\alpha\frac{\partial\delta^4}{\partial z} + \frac{1}{2}\cos\alpha\sin\varphi\frac{\partial\delta^4}{\partial\psi} + \frac{2}{3}\cos\alpha\cos\varphi\,\delta^4 = \frac{\mu k\Delta TD}{g\rho(\rho-\rho_v)\lambda_{fg}} \tag{15}$$

Introducing the dimensionless quantities,

$$\delta^* = \delta/\left[\frac{\mu k\Delta TD}{g\rho(\rho-\rho_v)\lambda_{fg}}\right]^{1/4} = \frac{\delta}{D}(R_a/Ja)^{1/4}, \text{ and } \quad z^* = z/D \tag{16}$$

into Eq. (15), one obtains the following dimensionless form for Eq. (15)

$$\sin\alpha \frac{\partial \delta^{+4}}{\partial z^{+}} + 2\cos\alpha \sin\varphi \frac{\partial \delta^{+4}}{\partial \psi} + \frac{8}{3} \cos\alpha \cos\varphi\, \delta^{+4} = 4 \qquad (17)$$

Eq. (17) is linear, nonhomogonous , partial differential equation of the first-order, which can be solved only numerically. At first, this equation will be solved for the case of being the tube in vertical position, and then for the horizontal case; and exact analytic solutions will be obtained. Finally, this equation will numerically be solved for a tube with an inclination angle α in the range : $0 \le \alpha \ge \pi/2$. This will next be presented in sequence.

2.1 Exact Solution for a Vertical Tube

For a vertical tube, one obtains an equation for the local film thickness by setting $\alpha = \pi/2$ in Eq. (17). This yields

$$\frac{d\delta^{+4}}{dz^{+}} = 4 \qquad (18)$$

Solution of Eq. (18) with zero thickness at the film start point of $z^{+} = 0$, is

$$\delta^{+} = (4z^{+})^{0.25} \qquad (19)$$

Using Eq. (19), dimensionless local heat transfer coefficient is found as

$$h^{+} = (4z^{+})^{-0.25} \qquad (19a)$$

wherein, it is introduced

$$h^{+} = h / \left[\frac{g\rho(\rho-\rho_v)k^3\lambda_{fg}}{\mu\, \Delta T\, D} \right]^{0.25} \qquad (19b)$$

The dimensionless average heat transfer coefficient is obtained using Eq. (19a) by

$$\bar{h}^{+} = 0.94\, (L^{+})^{-0.25} \qquad (19c)$$

wherein

$$\bar{h}^{+} = \bar{h} / \left[\frac{g\rho(\rho-\rho_v)k^3\lambda_{fg}}{\mu\, \Delta T\, D} \right]^{0.25} \quad \text{and} \quad L^{+} = L/D \qquad (19b)$$

Equation (19c) is just Nusselt solution for a vertical tube.

2.2 Exact Solution for a Horizontal Tube

Equation (17) with $\alpha = 0$, yields an equation for the loca film thickness along the periphery of a horizontal tube, as

$$\sin\varphi \frac{d\delta^{*4}}{d\varphi} + \frac{4}{3}\cos\varphi\, \delta^{*4} = 2 \qquad (20a$$

Setting $\varphi = 0$ in the above equation gives

$$\delta^{*}_{\varphi=0} = (3/2)^{1/4} = 1.11 \qquad (20b$$

which is the thickness of the condensate film at the top point o the horizontal tube. It is different from the zero value whic. was assumed by Nusselt. Wilson [9] studied the characteristic o transient, and steady film condensation on a horizontal tube. H found that the transient film becomes stable if the thickness a the top point of the horizontal tube ($\varphi=0$) reaches a certai value, called the critical thickness of the stable film, which i found by Eq. (1.2) in sec. 1. It is clear that the film thicknes atop a horizontal tube, found by Eq. (20b) from the presen analysis, is the same result obtained by Wilson from another typ of study. This confirms the present result.

Equation (20a) is linear, nonhomogonous, ordinar differential equation of first order, which can be solve analytically. This solution is found as

$$\delta^{*4}_{\varphi} = \delta^{*4}_{\varphi=0} + e^{-\int_0^{\varphi} s(\varphi)d\varphi} \left\{ \int_0^{\varphi} \left\{ G(\varphi)\, e^{\int_0^{\varphi} s(\varphi)d\varphi} \right\} d\varphi \right\} \qquad (21a.$$

wherein

$$s(\varphi) = \frac{4}{3}\cot\varphi \quad \text{and} \quad G(\varphi) = \frac{2}{\sin\varphi} \qquad (21b)$$

The values of the integral terms, involved in Eq. (21a), are calculated as follows

$$e^{-\int_0^{\varphi} s(\varphi)\, d\varphi} = \sin^{-\frac{4}{3}}\varphi \quad \text{and} \quad e^{\int_0^{\varphi} s(\varphi)\, d\varphi} = \sin^{\frac{4}{3}}\varphi \qquad (21c)$$

By inserting Eqs. (21b) and (21c) into Eq. (21a), and ther performing the involved integration, one obtains for the dimensionless local film thickness

$$\delta^{*}_{\varphi=0} = (1.5 + f(\varphi))^{1/4} \qquad (22a)$$

in which

$$f(\varphi) = 2\sin^{-\frac{4}{3}}\varphi \int_0^{\varphi} \sin^{\frac{1}{3}}\varphi\, d\varphi. \qquad (22b)$$

Using Eq. (22a), the dimensionless local heat transfer coefficient is determined by

$$h^*_\varphi = \frac{k}{\delta} = (1.5 + f(\varphi))^{-1/4} \tag{22c}$$

in which it is introduced

$$h^*_\varphi = h_\varphi \cdot \left[\frac{g\rho(\rho-\rho_v)k^3\lambda_{fg}}{\mu\ \Delta T\ D} \right]^{1/4} \tag{22d}$$

However, solution of Eq. (20a) with the boundary condition of zero film thickness at the top point of the horizontal tube (i.e., at φ=0) yields

$$\delta^*_\varphi = (f(\varphi))^{1/4} \tag{22e}$$

and

$$h^*_\varphi = (f(\varphi))^{-1/4} \tag{22f}$$

which is the same result found by Nusselt.

The function $f(\varphi)$ involved in both the Nusselt and present solution, has been calculated numerically by using the Simpson's method, as well by applying the trapezoidal formula. Both methods give the same results. Figures 2 and 3 display the present, and Nusselt solution for the peripheral distribution of the dimensionless local film thickness, and heat transfer coefficient, respectively. It is clear that the two solutions are similar in their general appearance, except at φ = 0 the present h^*_φ is finite and has a value of 0.99, while that of Nusselt goes to infinite value; due to Nusselt assumption of zero film thickness at φ=0. The difference between the two solutions decreases with increasing φ to be zero at $\varphi = \pi$. Hence, the average heat transfer coefficient can be calculated by integrating h^*_φ over the tube periphery as

$$\bar{h}^* = \frac{1}{\pi}\int_0^\pi h^*_\varphi \, d\varphi \tag{22h}$$

in which it is introduced

$$\bar{h}^* = \bar{h} \cdot \left[\frac{g\rho(\rho-\rho_v)k^3\lambda_{fg}}{\mu\ \Delta T\ D} \right]^{0.25} \tag{22i}$$

By inserting Eq. (22c) into Eq. (22h), and calculating numerically the integration, one obtains for the present solution

$$\bar{h}^*_\varphi = .65 \tag{23}$$

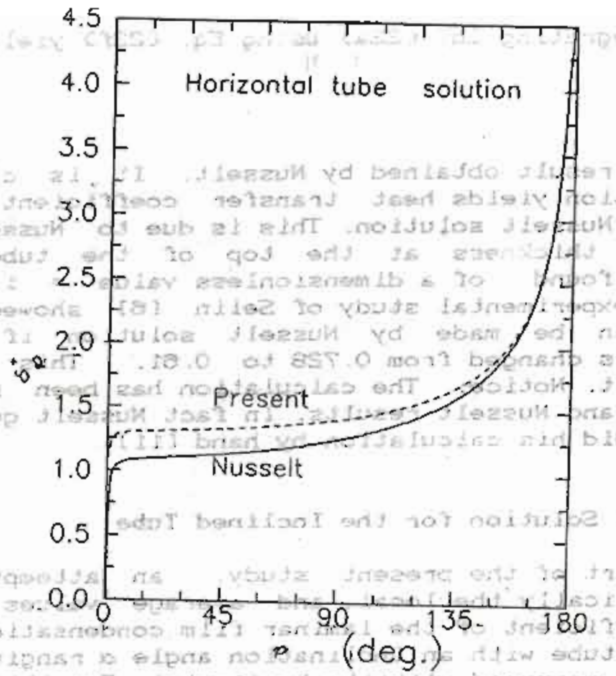

Fig. 2 The local condensate film thickness around the periphery of a horizontal tube.

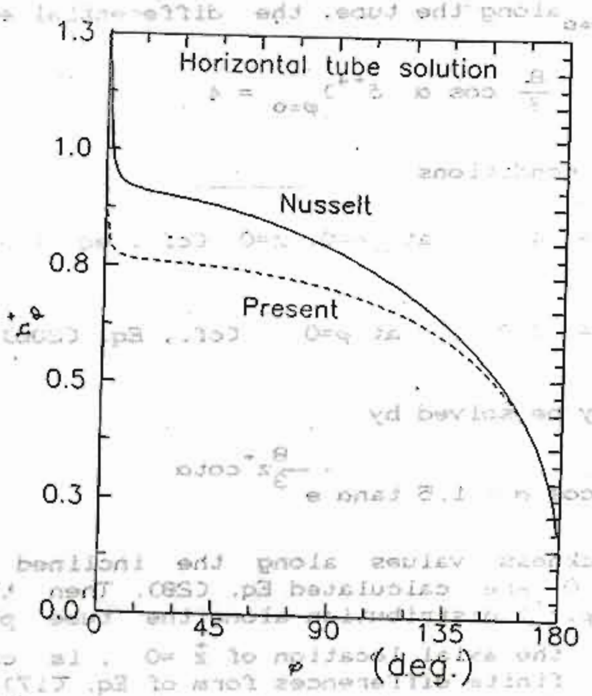

Fig. 3. The local heat transfer coefficient around the periphery of a horizontal tube.

However, integrating Eq. (22a) using Eq. (22f) yields

$$\bar{h}^{+}_{N} = 0.728 \qquad (24)$$

which is the result obtained by Nusselt. It is clear that the present solution yields heat transfer coefficient values lower than that of Nusselt solution. This is due to Nusselt assumption of zero film thickness at the top of the tube, while this thickness is found of a dimensionless value = 1.1 (cf., Eq. (20b)). The experimental study of Selin [6] showed that better prediction can be made by Nusselt solution if the constant coefficient is changed from 0.728 to 0.61. This confirms the present result. Notice: The calculation has been made for both the present, and Nusselt results. In fact Nusselt got a value of 0.724 as he did his calculation by hand [11].

2.3 Numerical Solution for the Inclined Tube

In this part of the present study, an attempt is made to predict numerically the local, and average values of the heat transfer coefficient of the laminar film condensation on the outer surface of a tube with an inclination angle α ranging from 0° to 90°, which is measured with the horizontal. For this purpose, the local film thickness δ^+ will numerically be calculated by solving the partial differential equation (17). The numerical procedure applied, is described in the following steps :

1- With $\varphi = 0$ in Eq. (17), one obtains for the local film thickness $\delta^+_{\varphi=0}$ along the tube, the differential equation :

$$\left(\sin\alpha \frac{d\delta^{+4}}{dz^+} + \frac{8}{3}\cos\alpha\ \delta^{+4}\right)_{\varphi=0} = 4 \qquad (26)$$

which with the conditions

$$\left.\frac{d\delta^{+4}}{dz^+}\right|_{\alpha=\pi/2} = 4 \quad \text{at} \quad \varphi=0,\ z=0 \quad (\text{cf., eq.}(18)) \qquad (27a)$$

$$\left.\delta^{+4}_{\varphi=0}\right|_{\alpha=0} = 3/2 \quad \text{at } \varphi=0 \quad (\text{cf., Eq. } (20b) \qquad (27b)$$

can analytically be solved by

$$\delta^{+4}_{\varphi=0,z} = 1.5/\cos\alpha - 1.5\tan\alpha\ e^{-\frac{8}{3}z^{+}\cot\alpha} \qquad (28)$$

2-The film thickness values along the inclined tube at the points of $\varphi = 0$ are calculated Eq. (28). Then the local film thickness $\delta^+(\varphi,z^+)$ distribution along the tube periphery from $\varphi = 0$ to π, at the axial location of $\dot{z} = 0$, is calculated from the backward finite differences form of Eq. (17), by applying one of the known numerical methods such as Euler method or Runge-Kutta procedure. Both techniques are found to give approximately very close results.

3-Step 2 is repeated at the next axial locations $z^+ = j\ \Delta z^+$; j=1,2, ..., M. The computation process will proceed in z-direction until $z^* = L^*$. Grid with $\Delta z^* = \Delta\varphi$ is used. The number of grid points in peripheral direction was 60.

4-Then, the local peripherally-averaged heat transfer coefficient at any axial location, $z_j^* = j\Delta z$, is calculated by

$$h^*(z_j^*) = \frac{k}{\delta^*(z_j)} = \frac{k}{\pi}\int_0^{\pi} \frac{1}{\delta^*(\varphi, z_j^*)}\, d\varphi \qquad (29)$$

5-Finally, overall average heat transfer coefficient is calculated for total length L^* by

$$\bar{h}^* = \frac{1}{L^*}\int_0^{L^*} h^*(z_j^*)\, dz^* \qquad (30)$$

The above integrations are numerically calculated by employing Simpson's formula. The accuracy of the numerical results has been checked up by comparison the numerical results of the horizontal, and that of the vertical tube with the corresponding known exact solution.

2.3.1 Results and Discussion

Figure 4 displays the value of the dimensionless film thickness at the top point ($\varphi=0, z^*=0$) of a tube; calculated from Eq. (28) with $z^* = 0$, versus the tube inclination angle α with the horizontal. It is clear that for the horizontal case ($\alpha = 0$), the film thickness at the top point, is of a dimensionless value = 1.1, while is of zero value for the vertical case ($\alpha = \pi/2$). It is also evident that with increasing α, the film thickness continuously decreases to fall to zero when $\alpha = \pi/2$.

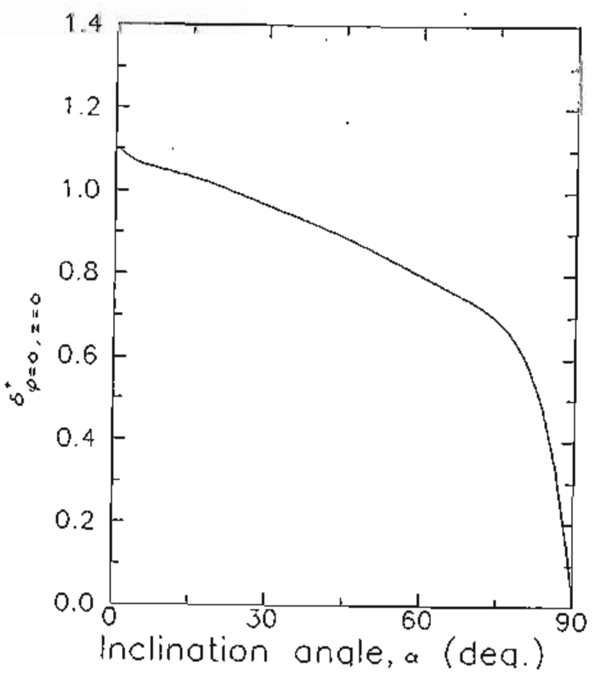

Fig. 4 Thickness of the condensate film at the top point of a tube as function the inclination angle with the horizontal; $0° \le \alpha \le 90°$.

Figures 5 and 6 show the peripheral variation of the local film thickness $\delta^{+}(\varphi,z^{+})$, and the heat transfer coefficient $h^{+}(\varphi,z^{+})$, respectively; at three different axial locations. As it can be seen from the curves, near the film start; at $z^{+} = 2$, the film is thin. However, at higher axial location, the film is thick, and becomes thicker in the peripheral direction to be of maximum thickness at $\varphi = \pi$. This results in local heat transfer coefficient $h(\varphi, z^{+})$ decreases in both the axial, and the peripheral direction.

Figures 7 shows the axial variation of the local peripherally- averaged heat transfer coefficient $h^{+}(z^{+})$, calculated from Eq. (29), for different inclination angles. It is clear from the results that for small inclination angle $\alpha \leq 15^{\circ}$, $h^{+}(z^{+})$ exceeds the value of the horizontal case over the entire tube length of $z^{+} = 60$. However, for $\alpha > 15^{\circ}$, $h^{+}(z^{+})$ is below the horizontal value over the tube length, except for a short length at the upper part of the inclined tube, and an increase in α results in a decrease in the heat transfer coefficient. This decrease in $h^{+}(z^{+})$ is more pronounced for change in α from 85° to 90°.

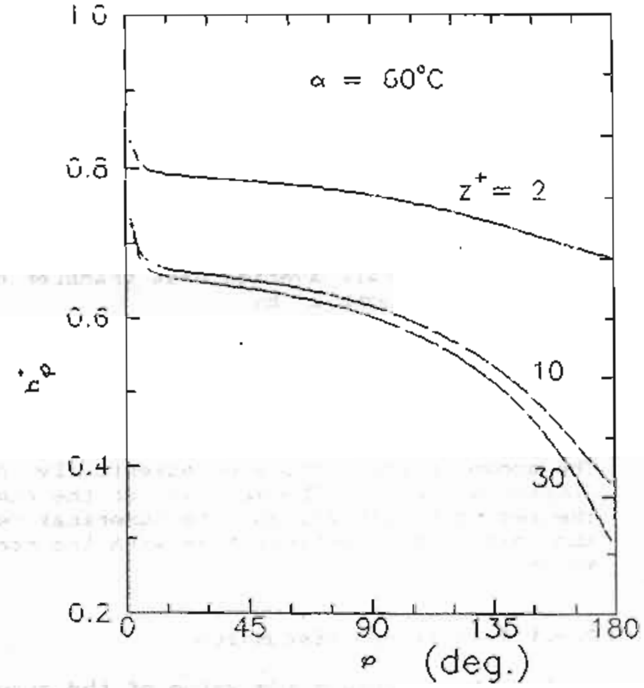

Fig. 5 Variation of the local heat transfer coefficient around the periphery of an inclined tube at various axial locations.

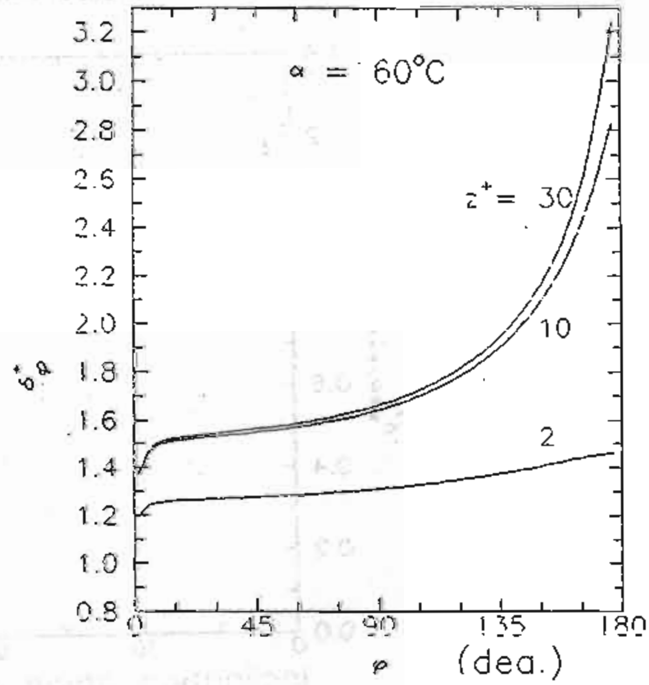

Fig. 6 The peripheral Variation of the local film thickness at different axial locations.

This means that for a vertical tube, an inclination angle of about 5° from the vertical position, results in an increase in the heat transfer coefficient of about 50%. This behavior of $h^+(z^+)$ is attributed to the inversely behavior of the local peripherally- averaged film thickness $\delta^+(z^+)$ displayed in Fig. 8, which may be attributed to the effect of change in the gravity force components; in the peripheral, and in the axial direction, due to change in the tube inclination angle. Since the laminar film of condensing stagnant vapour is mainly driven on the tube surface by the gradational force.

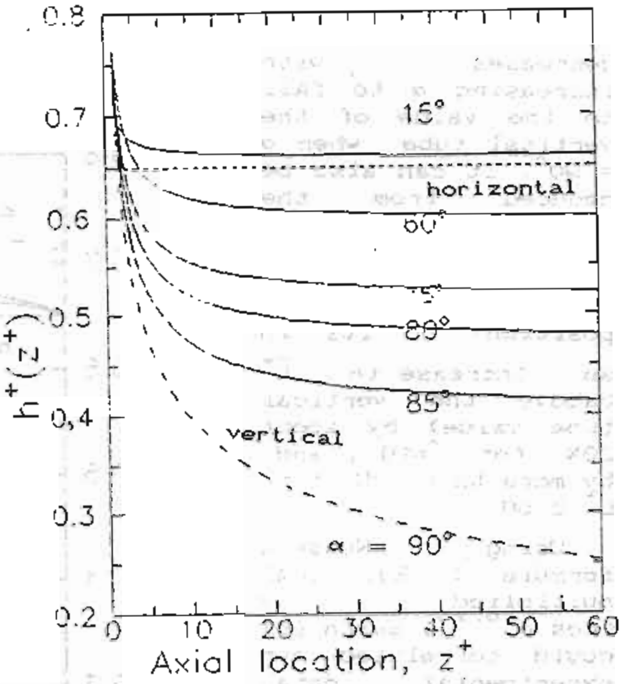

Fig. 7 The axial variation of the local periphery-averaged heat transfer coefficient at various axial locations.

Figures 9 shows variation of the overall average heat transfer coefficient $h^+(L^+)$; numerically calculated using Eq. (30), versus the tube inclination angle, for two different values of the tube dimensions ratio L^+. It can be seen that for $\alpha = 0$, the numerical solution gives the exact value of the horizontal tube (cf., Eq. (23)), while for $\alpha = 90^\circ$ yields the exact value of the vertical tube (cf., Eq. (19c)). It is also clear that for an increase in α from zero up to a certain value dependent on the magnitude of L^+, the overall heat transfer coefficient exceeds the horizontal tube

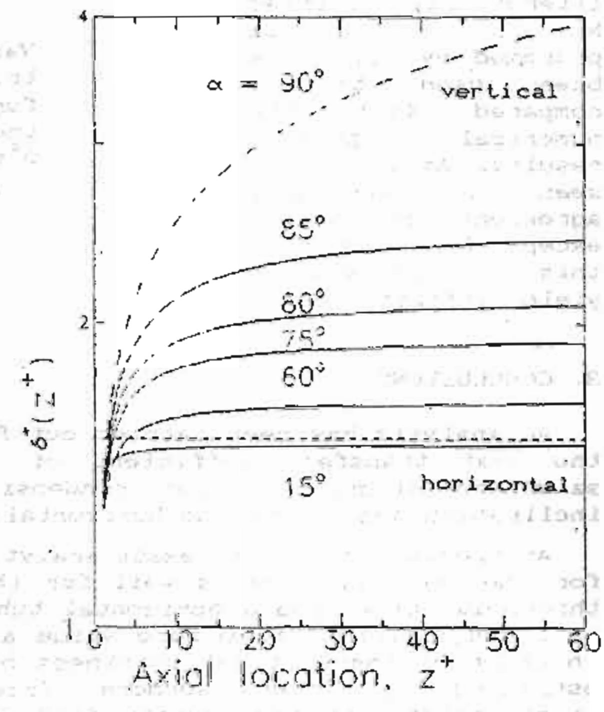

Fig. 8 The axial variation of the local periphery-averaged film thickness at different axial locations.

value, then it decreases with increasing α to fall to the value of the vertical tube when $\alpha = 90^{\circ}$. It can also be deduced from the results that the inclination of a vertical tube by 5° from its vertical position results in an increase in $\bar{h}^{+}$ (above the vertical tube value) by about 20% for $L^{+}=10$, and by more than 50% for $L^{+} \geq 60$.

Using Nusselt formula (Eq. (24) multiplied by $(\cos \alpha)^{0.25}$, Selin [6] could correlated own experimental data, obtained for $0 \leq \alpha \leq 60^{\circ}$, within ± 15%. As neither another correlation nor experimental data are available to us from literatures, modified Nusselt formula proposed by Selin has been used to be compared with the numerical present results. As it can be seen, reasonable agreement is found, except for $\alpha = \pi/2$, this correlation yields infinite value.

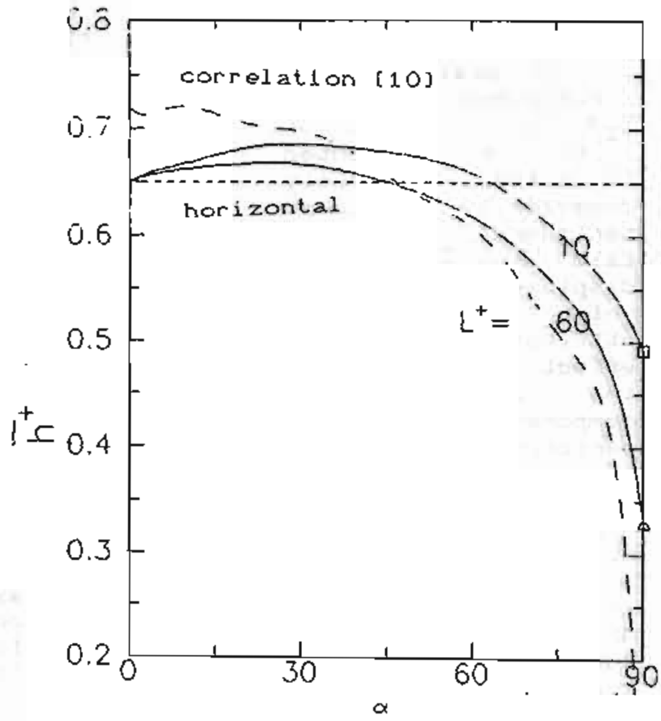

Fig. 9 Variation of overall heat transfer coefficient as function of the tube inclination angle ranged from 0° to 90°.

3. CONCLUSIONS

An analysis has been carried out for the purpose of predicting the heat transfer coefficient of the laminar film for pure saturated stagnant vapour condensing outside a tube of an inclination angle with the horizontal; in the range from 0° to 90°.

As special solutions, exact analytic solution has been obtained for the vertical tube as well for the horizontal tube. The film thickness value atop a horizontal tube has been determined, which is found different from zero value assumed by Nusselt, but it is in order of the critical thickness of the stable film, which is estimated by other authors from studying the stability characteristic of the laminar film of saturated stagnant vapour condensing on a horizontal tube. Consequently, a modification in Nusselt solution has been resulted; the lead constant is changed from 0.728; as found by Nusselt assuming zero film thickness atop the tube, to 0.65 in the present solution. Such a modification was also proposed by other authors to achieve better agreement with experimental data. This confirms the present theoretical modification.

Numerical solution has been performed for the inclined tube to predict local, and average values of the heat transfer coefficient. Results have been obtained for inclination angles from 0° to 90°. No assumption are made for the film thickness values at the top points (of $\varphi=0$, cf. Fig. 1) along the inclined tube, but they are exactly determined by Eq. (28). The exact solution of the horizontal, and that of the vertical tube are used to check up the accuracy as well validity of the numerical solution.

Present results show that small inclination angle from the horizontal, yields improved heat transfer coefficient with respect to the horizontal value. While inclination of a vertical tube by about 5° from the vertical position, results in an increase in the heat transfer coefficient more than 60% (above the vertical value) for a tube dimensions ratio, $L/D \geq 60$, and the magnitude of this increase is lower for smaller L/D ratio.

NOMENCLATURE

C_p	specific heat at constant pressure
D	tube diameter
g	acceleration of gravity
$\bar{h}^{+}$	dimensionless average heat transfer coefficient
h^{+}_{φ}	dimensionless local heat transfer coefficient in the peripheral direction of the tube
L	tube length
L^{+}	dimensionless tube length ratio, L/D
$h(z^{+})$	dimensionless local periphery-averaged heat transfer coefficient in the axial direction of the tube
Ja	Jacob or phase change number, $C_p(T_s-T_v)/\lambda_{fg}$
k	thermal conductivity
M	number of the grid points in the axial-direction
m''_c	condensation mass flux through liquid-vapour interface
Pr	Prandtl number, $C_p\mu/k$
R_a	Raleigh number, $g(\rho-\rho_v)PrD^3/\rho\nu^2$
T_w	wall temperature
T_s	vapour saturation temperature
ΔT	temperature difference, T_s-T_w
u, v, ω	velocity components in x, y and z directions, respectively
x	tangential coordinate
y	radial coordinate
z	axial coordinate

z^{+i}	dimensionless axial location, z/D
Δz^{+}	grid size in z-direction

Greek symbols

α	tube inclination angle with the horizontal
δ^{+}	dimensionless film thickness
ν	kinematic viscosity
μ	dynamic viscosity
ρ	liquid density
φ	peripheral angle
λ_{fg}	latent heat of condensation

Subscripts

v	vapour
c	condensation

Superscripts

+	dimensionless
j	index of grid points number in z-direction
p	present
N	Nusselt
*	modified

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