# FLOW OF DILUTE POLYMER SOLUTIONS
## IN POROUS MEDIUM

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ABSTRACT

The flow of dilute polyacrylamide solutions in a porous media of
non spherical particles is studied. Different particle sizes and poly-
mer concentrations are used. The results show that undetectable con-
centrations as 2 wppm causes a substantial increase in the flow resis-
tance of porous medium. Such non Newtonian effects is attributed to
the increase in elongational viscosity which is found to characterize
polymer solutions. Elongational flow fields are almost found in por-
ous media and in the near wall region of turbulent boundary layers.
Experimental data show that the increase in flow resistance due to
polymer additives is proportional to $(c[\mu])^n$ where $n = 0.75$.

NOMENCLATURE

| | |
|---|---|
| A | aspect ratio of elongated polymer molecule |
| c | polymer concentration, gm/c.c |
| $d_1$ | lower limit of particle size range, mm |
| $d_2$ | uper limit of particle size range, mm |
| D | diameter of the porous section, mm |
| $D_p$ | particle diameter, mm |
| $f_P$ | friction (resistance) factor of porous medium; $f = \dfrac{\Delta P/1}{\varsigma u_o^2} D_p \dfrac{\varepsilon^3}{1-\varepsilon}$ |
| G | elongational strain rate; $G = du/dx$; $sec^{-1}$ |
| k | constant; eqn. (12). |
| K | permeability of the porous medium given by Darcys law |
| L | length of porous medium section; mm |
| M | molecular weight of polymer |
| n,N | exponent and constant of eqn. (18) |
| p | static pressure; $N/m^2$ |
| $\Delta P$ | pressure drop along the porous section; $N/m^2$ |
| Q | volume flow rate; $m^3/sec.$ |
| R | pipe raidus; mm |
| $R_h$ | hydraulic radius; mm |
| Re | Reynolds number; $Re = \dfrac{u_o D_p \varsigma}{\mu} \cdot \dfrac{1}{1-\varepsilon}$ |
| u | streamwise velocity, m/sec. |
| $u_m$ | average streamwise velocity in the capillary tube; m/sec. |
| $u_o$ | superficial velocity; $u_o = Q/((\pi/4) D^2)$m/sec. |
| x | streamwise direction |
| z | constant; eqn. (1a) |
| $\alpha$ | viscous term geometry parameter; eqn. (11) |
| $\beta$ | form drag term geometry parameter; eqn. (11) |
| $\gamma$ | elastic term geometry parameter; eqn. (11) |
| $\varepsilon$ | porosity of the medium |
| $\eta$ | extensional viscosity |
| $\lambda$ | relaxation time of polymer solution; sec. |
| $\mu$ | dynamic viscosity |
| $[\mu]$ | intrinsic viscosity; c.c/gm |
| $\varsigma$ | density. kg/m |
| $\sigma$ | normal stress in streamwise direction; $N/m^2$ |
| $\phi$ | constant; eqn. (15) |
| $\gamma$ | polymer parameter; eqn. (16). |

## 1, INTRODUCTION

Flow through porous media is of great importance because of its many industrial and engineering applications such as filtration processes, chemical industry and packed bed's technology. In fact many industrial processes involve non Newtonian fluid flows through porous medium as in oil and chemical industries. From the point of view of drag reduction, the flow of non Newtonian polymer solutions in porous medium is very intresting due to the fact that the flowing fluid is subjected to acceleration as well as deceleration. Under such flow configuration polymer molecules are subjected to stretching at sufficiently high strain rates and elastic effects appear. It has been postulated that the reduction in frictional drag of turbulent shear flows is caused by the high resistance to stretching which characterize polymer solutions. It inhibits the stretching of the vortex structure in the near wall region which delays the development and ejection phases of the bursting cycle [1]. By doing so, the frequency of turbulent burst ejections from the wall decreases compared with that of Newtonian fluid flow. Thus, the generation of turbulence is decreased and subsequently the frictional drag is reduced. Accordingly, the study of the behaviour of drag reducing additive fluid flows in extensional flow fields can present valiable information to explain the phenomenon as well as supplying data for practical applications.

Most of the studies were concerned with investigating elongational flows of concentrated polymer solutions. Flow fields such as in expanding jet, through orifice, between cylinderical rollers and through porous medium were experimentally studied. In fact, concentrated polymer solutions exhibit non Newtonian behaviour even in simple shear flow. Although these investigations are important for engineering applications, they are of little benefit to drag reduction. Experiments and analyses of concentrated polymer solutions in such elongational flow fields were reviewed by Savins [2] . The work of Dauben and Menzie [3] was the first to study the flow of dilute polymer solutions through porous medium. This was followed by the work of James and McLaren [4] . Experiments similar to theirs were carried out by Elata et al. [5] and Naudascher et al. [6] . Elata et al. [5] carried out their experiments using solutions of different concentrations of polyox coagulant flowing through porous beds of spherical particles under laminar flow conditions. Laufer et al. [7] studied the flow behaviour of two dilute polymer solutions, Polyox WSR 301 and Separan AP 273, at concentrations as low as 25 wppm through porous beds of spherical particles. The results showed that polymer additives increase the pressure drop by 2-10 times compared with Newtonian one. This is attributed to the increase of stretching resistance. Although the scatter of the experimental data available is wide, the agreement in trend is clear.

The aim of this work is to study the flow of dilute polyacrylamide solutions through porous medium of irregular shape particles (sand) under laminar, transition and turbulent flow conditions.

## THEORETICAL BACKGROUND

The discription of a Newtonian fluid through porous medium is based on the classical experiment of Darcy's Law for one dimensional flow [8] .

$$u_o = \frac{K}{\mu} \frac{\Delta P}{L} ; \qquad\qquad \dots\dots(1)$$

$u_o$ is the superficial velocity, $\mu$ is the dynamic viscosity and $K$ is the permeability of the porous medium. The quantity $K$ depends on the structure of the porous medium as well as the flow regime.

The most common model for the flow in a porous medium is that known as the capillary model. In this model, the medium is represented by a solid permeated by an assemblage of tortuous, continuous capillaries, which have non uniform cross sections. For such capillaries, the mean hydraulic radius is related to the particle diameter $D_p$ and the void fraction (porosity) $\varepsilon$ as

$$R_h = \frac{D_p}{6} \frac{\varepsilon}{1-\varepsilon} \qquad\qquad \dots\dots(2)$$

and the average velocity of the flow through a capillary is related to the superficial velocity $u_o$ by

$$u_a = u_o / \varepsilon \qquad\qquad \dots\dots(3)$$

For laminar flow through a pipe of radius $R$, Hagen-Poiseuille gave the pressure drop $\Delta P$ as

$$\Delta P = 8 \mu u_a L/R^2 \qquad\qquad \dots\dots(4)$$

Regarding the porous medium as a conduit with complicated cross section the mean hydraulic raduis $R_h$ given by eqn.(2) is related to $R$ by $R_h = R/2$. Using the superficial velocity $u_o$, the pressure drop for laminar flow through packed bed can be written as

$$\Delta P = 72 \frac{u_o \mu L}{D_p^2} \frac{(1-\varepsilon)^2}{\varepsilon^3} \qquad\qquad \dots\dots(5)$$

So far, we have considered the length of the capillary to be that of the bed depth which is incorrect since the fluid must flow through a tortuous pass of greater length. Experimental data showed that the constant in equation (5) must be 150 instead of 72, giveing what is known as Blake-Kozeny equation. Assuming that the friction factor "f" and Reynolds number "Re" of a packed bed are defined as

$$f = \frac{\Delta P/L}{\rho u_o^2} D_p \frac{\varepsilon^3}{1-\varepsilon} \quad , \qquad Re = \frac{\rho u_o D_p}{\mu} \frac{1}{1-\varepsilon}$$

Blake-Kozeny equation can be written as;

$$f = 150/Re \qquad\qquad \dots\dots(6)$$

This is valied for laminar flow regime where $Re \leqslant 10$. For highly turbulent flow where $Re > 1,000$, experimental data show that packed bed friction factor has a constant value giving what is known as Barke-Plummer equation as

$$f = 1.75 \qquad\qquad \dots\dots(7)$$

For the flow in the transition region; $10 < Re < 1,000$, Ergun[9] shows that the experimental data of several investigators correlates very well by simply adding laminar and turbulent expressions as;

$$f = 150/Re + 1.75 \qquad\qquad \dots\dots(8)$$

This relation which is known as Ergun equation represents the flow in laminar, transition and turbulent regimes and fits available experimental data of Newtonian fluid flows very well.  The advantages of this equation is that it can be written in the form

$$P = \alpha \mu u_0 + \beta \rho u_0^2 \qquad \ldots\ldots(9)$$

in which the parameters $\alpha$ and $\beta$ depend only on the geometery of the porous medium ($D_0$ and $\varepsilon$ ).  This form of Ergun equation is interpreted as that the total resistance consists of a viscous part ($\alpha \mu u_0$) and a form resistance part ($\beta \rho u_0^2$).  Such form of Ergun equation can be easly modified to account for the viscoelastic characteristics of dilute polymer solutions.

As the fluid flows through a porous medium, it encounters continuous contractions and expansions and elongational stresses appear. In such flow situations, elastic and non-Newtonian effects of dilute polymer solutions occur when the relaxation time $\lambda$ of the fluid exceeds the time scale of the flow.  The fluid, then, will not accommodate the flow changes and an increase in the flow resistance will be noted.  Such increase is interpreted as an increase in the elongational viscosity due to stretched polymer molecules.  For a Newtonian fluid contains dissolved or suspended material, an additional stress term may be generated. Therefore, the normal stress in the mean flow direction can be expressed as

$$\sigma_{11} = -P + (2 \mu + \eta )G \qquad \ldots\ldots(10)$$

where P is the isotropic pressure, G is the streamwise strain rate (du/dx) and $\eta$ is the elongational viscosity contributed by the presence of the polymer molecules in the flow.  From that equation. it is clear that the increase in flow resistance of porous media due to elastic and non Newtonian effects is proportional to $\eta$ G.  The elongational strain G is proportional to the superficial velocity $u_0$. Equation (9) can be modified to include the elastic and non Newtonian effects as

$$\Delta P = \alpha \mu u_0 + \beta \rho u_0^2 + \gamma \eta u_0 \qquad \ldots\ldots(11)$$

As ($\alpha \mu u_0$) represent the viscous contribution and ($\beta \rho u_0^2$) the form drag contribution, ($\gamma \eta u_0$) represent the elastic and non Newtonian contribution in the total resistance; where $\gamma$ is another parameter that depends upon the porous media's geometery ($\varepsilon$ , $D_0$). Accordingly, the friction factor of viscoelastic non Newtonian fluid flow in porous media can be written as

$$f = \left[150 + k (\eta/\mu)\right] /Re + 1.75 \qquad \ldots\ldots(12)$$

where k is a numberical constant.

The elongational viscosity $\eta$ of polymer solutions rapidly increases with the elongational deformation rate, then tends to saturate at certain large value when the elongation rate exceeds the reciprocal of the relaxation time $\lambda$ .  It is convenient to use the elongational viscosity relationship derived by Batchelor [10] for elongated particles of aspect ratio "A" as

$$\eta / \mu = \frac{4}{3} c A^2/\ln ( \pi/c) \qquad \ldots\ldots(13)$$

where c is the volume concentration of polymer molecules which is taken as the mass concentration since the specific gravity of the polymer is 1.0. The aspect ratio of the elongated polymer molecules "A" depends on the strain rate G of the flow. It starts to grow when the strain rate G becomes large enough to stretch the polymer molecules. Further increase in the strain rate G causes more stretching of the molecules which increases A and hence the non Newtonian effects i.e., A will be proportional to $\eta$G. As A continues to grow with the strain rate $\eta$ will increase until it saturates when A reaches a limiting value at which polymer molecules are fully extended. Hence, becomes constant which can be approximated as

$$\eta/\mu = Z \, c \, [\mu] \, /\ln(\pi/c) \qquad \qquad \ldots\ldots(14)$$

where $[\mu]$ is the intrinsic viscosity which is given by

$$[\mu] = K \, M^{0.78} \; ; \quad K = 1.25 \times 10^{-2}$$

and Z is a constant that depends on polymer type, for polyacrylamide water solution with $M = 5 \times 10^6$, Z has a value of $3 \times 10^5$[11]. Using the expression given by equation (14), the friction factor of packed bed polyacrylamide flow can be given as

$$f = \left[150 + \emptyset \, c \, [\mu] \, /\ln(\pi/c)\right] \, /R + 1.75 \ \ldots\ldots(15)$$

where $\emptyset$ is a constant; $(\emptyset = kZ)$ that can be determined experimentally. Since $\ln(\pi/c)$ does not vary appreciably within the dilute concentration range of our intrest, it will be considered as constant value and will be included in the constant $\phi$, $(\phi = kZ/\ln(\pi/c))$. Hence equation (15) becomes

$$f = [150 + \psi]/Re + 1.75 \qquad \qquad \ldots\ldots(16)$$

$$\psi = \phi \cdot c \, [\mu] \qquad \qquad \ldots\ldots(16\text{-}a)$$

where $\psi$ is a parameter that depends upon polymer type and concentration.

## 3. EXPERIMENTAL TECHNIQUE

Experimental rig is shown in figure (1). It is a gravitational open-flow system. The fluid stored in an overhead tank, 200 liters capacity supplies a small constant head overflow tank. From which the fluid flows under gravity action through the test section, then, to the atmosphere and discarded. The test section is a porous bed of sand particles contained by two screens in a 28 mm diameter tube of 150 mm length. The porous section is kept horizontal at a level of 6,000 mm down that of the fluid in the overflow tank. The pressure drop along the porous section $\Delta P$ is measured using a U tube mercury manometer connected to two pressure taps 1.0 mm diameter located just upstream and downstream the test section. The flow rate is measured by the time taken by the discharged fluid to fill a volume of 400 cm$^3$.

Throughout this work, five porous sections made of different particle sizes are used. Each porous section is made of sand particles with diameters limited to a sertain range of sizes. Five different ranges of sand particle sizes are used to make the different porous sections as (0.25 - 0.625)mm, (0.625 - 1.0)mm, (1.0 - 1.25)mm, (1.25 - 2.0)mm and (2.0 - 4.0)mm respectively. As the sizes of the sand particles in each range are randomly distributed across the end limits of the range, it is convenient to consider the average particle diameter $D_p$ to be
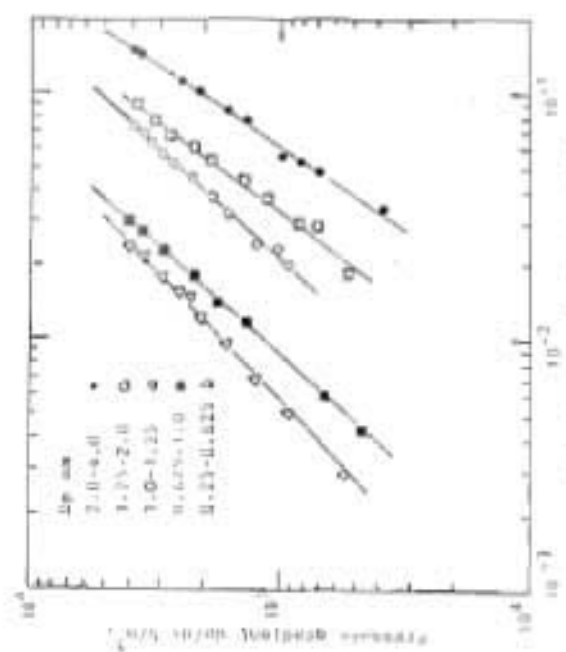
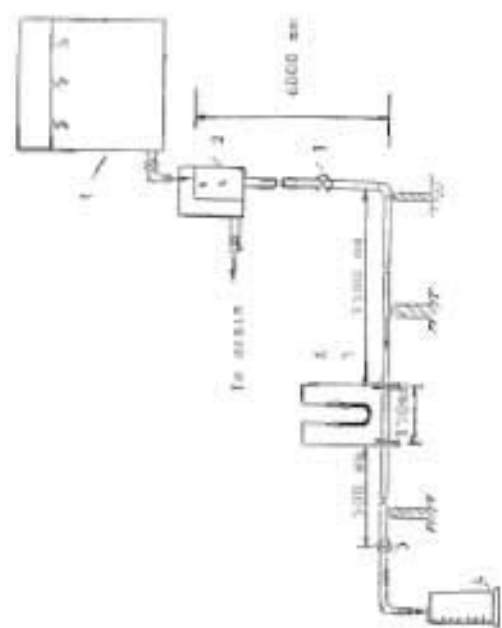Fig.(2) Pressure gradient $dp/ds$ versus superficial velocity $V_0$ for solar flow.



Fig.(1) Experimental rig:
1- Head tank, 2- constant head tank, 3- control valve,
4- test vessel,5- test section, 6- U tube mercury manometer.
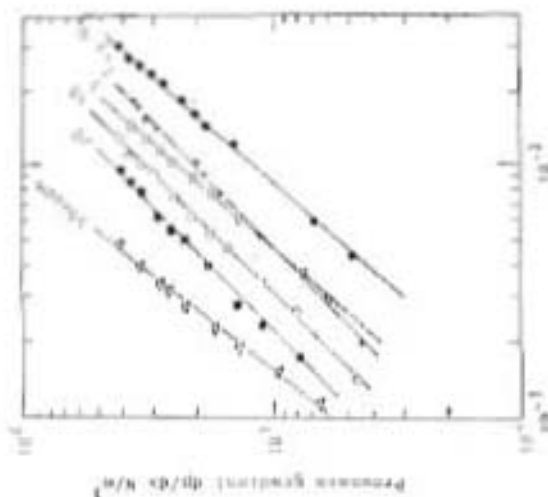
Fig.(4) Typical results of pressure gradient dp/dx of pollutants-fluids solutions versus Reynold's number compared with water film for $D_p = 0.625-1.0$ mm.
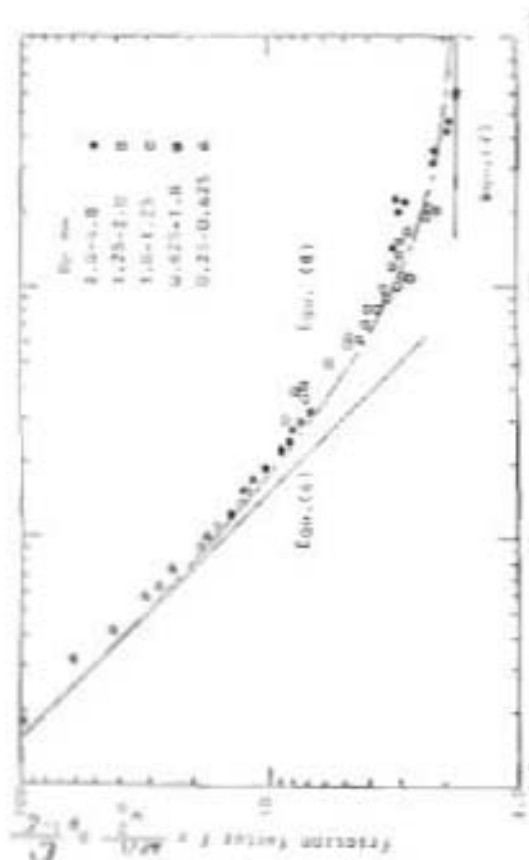


Fig.(5) Friction factor of fluids versus x versus Reynolds number $Re_f/2$ water film.

Fig.(6) Friction factor of polyacrylamide solutions of
different concentrations; 5,20 wppm respectively;
through porous media as function of Reynolds number.

——— water flow
——— predicted results for $\psi = h(c(\mu))^{0.75}$
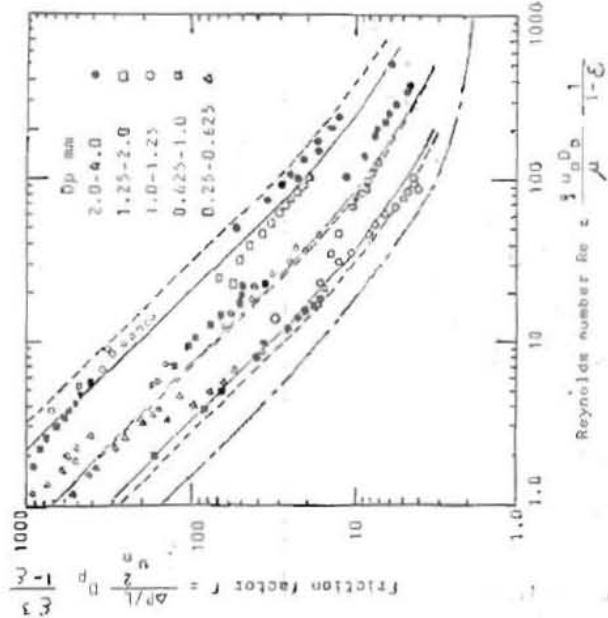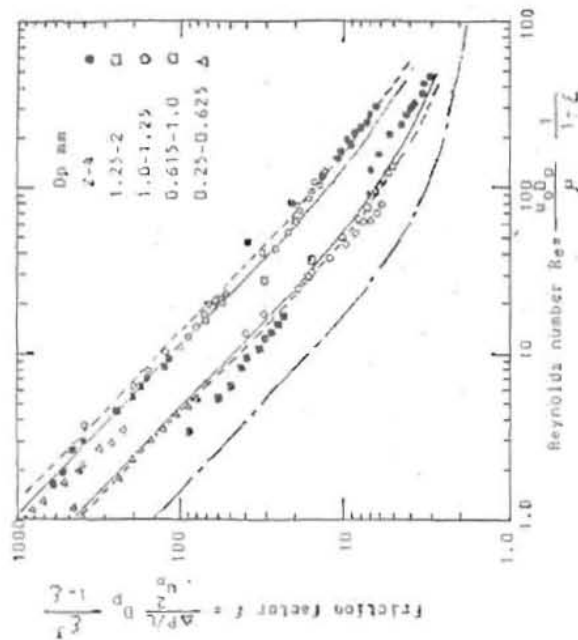———— predicted results for $\psi = \beta \, c(\mu)$.



Fig.(5) Friction factor of polyacrylamide solutions at
different concentrations; 2,10 and 50 wppm res-
pectively through porous media as function of
Reynolds number.

——— water flow
——— predicted values for $\gamma = h(c(\mu))^{0.75}$
———— predicted values for $\gamma = \beta \, c(\mu)$

$$D_p = (d_1 + d_2)/2$$

where $d_1$ and $d_2$ are the lower and uper limits of the particle dia-
meters in the range considered. This gives an average particle dia-
meter in each of the five porous medium sections as 0.4375, 0.8125,
1.125, 1.625 and 3.0 mm respectively. The porosity $\varepsilon$ of each section
is measured experimentally by the volume of water that just covers
the dried sand in a certain volume (100 cm$^3$). For each section at
least 10 experiments were carried out and averaged to give a mean
value for "$\varepsilon$".

The experimental work was made by carrying out a series of measu-
rements of pressure drop and flow rate for each porous section using
either water flow or dilute polyacrylamide solutions at different con-
centrations . Five concentrations; 2,5,10,20 and 50 wppm were used
throughout this work.

## 4. RESULTS AND DISCUSSION

### 1- Water Flow Results:

In order to check the performance of the experimental rig and
the measuring equipment, a series of experimental tests were carried
out for pure water flow through the five porous sections. In addit-
ion, water flow results are also used for the comparison with dilute
solutions of water-polyacrylamide to detect the changes due to the
presence of additives in the flow. Typical water flow results of
the pressure gradient dp/dx as a function of the superficial velocity
$u_0$ are shown in figure (2). It is more convenient to plot these
results in the nondimensional Ergun coordenatcs relating the friction
factor of packed bed f and Reynolds number Re as defined earlier.
The particle diameter $D_p$ is taken as the mean value between the lower
and higher limits of the particle size distribution. The porosity of
the medium is experimentally determined with values varying from
0.24 for wider range of particle rize to 0.33 for smaller one.
Such presentation of results made all experimental data to collapse
on one curve as shown in figure (3). Comparing these results with
those reported by Ergun and represented by the empirical formula;

$$f = 150/Re + 1.75$$

show excellent agreement. The experimental results reported in this
work confirm the fact that Ergun equation is also valied for porous
media of non spherical particles, in spite of the scatter found at high
Reynolds number (large size particles).

### 2- Dilute Polymer Solutions:

Five different dilute concentrations of polyacrylamide-water
solutions were tested in each of the five porous sections; they were
2,5,10,20 and 50 wppm. Typical pressure gradient dp/dx results
versus the superficial velocity $u_0$ for the flow of dilute polymer
solutions through the 0.625 - 1.0 mm particle size porous section
are shown in figure (4) in comparison with water flow results. They
show remarkable increase in dp/dx for the same flow rate ($u_0$) compa-
red with water flow results. As shown in figure (4), 5 wppm of
polyacrylamide causes an increase of about 100% in the pressure gra-
dient. 50 wppm polymer solution increases the filtration resistance
($\Delta P/L$) by almost 10 times that of the solvent (Newtonian) fluid

flow. Such very low polymer concentrations, which have no detectable effects on the physical properties such as the shear viscosity and the density of the solvent, can cause serious problems to filtration processes. As it has been shown in figure (3) that the results are best to be represented in the non-dimensional Ergun coordenates f and Re. Friction factor results of the flow of dilute concentrations of polyacrylamide as function of Reynolds number are shown in figures(5) and (6). Figure (5) presents the results of 2,10 and 50 wppm solut-ions in comparison with those of the solvent (water), while figure(6) presents the resistance factor f results for 5 and 20 wppm concentra-tions. In general, the results show that the presence of undetectable concentrations of high molecular weight polymers, causes large increa-se in the flow resistance of a porous media when compared with that of the solvent (Newtonian) flow. Such non Newtonian effect increases with the concentration of the polymer. It may be seen from these two figures that the increase in the hydrodynamic resistance of the flow covers the whole range of Reynolds number investigated. Howver, the results show no sign of onset point as that reported by Elata et al. [5] , James et al. [6] and Naudascher et al.[6] . In fact, this is due to the fact that the lowest strain rate of the flow developed in this work is higher than that of the onset point.

As it has been discussed before, the flow through porous medium is actually a flow in a tortous channel with variable cross section such that the flowing fluid is subjected to continuous expansions and contractions. In such situations extensional flow exists. It has been postulated that linear high molecular weight polymer solutions exhibit substential increase in extensional viscosity even for very dilute concentrations. This is due to stretched polymer molecules in flow direction. In all reological models, extensional viscosity is directly related to additive concentration and elongational strain rate. When polymer molecules are fully extended (strectched), exten-sional viscosity becomes solely dependent on concentration. Batchelor's modle [10] assumed that $\eta$ is proportional to $c\,[\mu]$ . Accordingly, the increase in resistance coefficient $\Delta f$ due to polymer additives can be written, using equation (16), as

$$\Delta f \;.\; Re = \gamma \qquad\qquad \ldots\ldots(17)$$

This relation show that the increase in flow resistivity (inverse premeabillity) of a porous medium due to polymer additives which is defined as $\Delta(Re.f)$ is a pure function of the concentration and inde-pendence of the flow rate. This had been proved when $\Delta(Re.f)$ is plo-tted in figure (7) as function of Reynolds number for the different polymer concentrations tested. In spite of the scatter found, the results of $\Delta(Re.f)$ clearly show constant value independence of Re. In fact this scatter is found in the results of different porous sections. which is naturally due to experimental errors in measuring porosity $\varepsilon$ , particle sizes, different ages of tested solutions,..etc. On the other hand, it is difficult to find remarkable scatter in the results of the same test section due to the constancy of these parameters.

For each polymer concentration, $\Delta(Re.f)$ are averaged giving a mean value $\Delta(Re.f)m$. These mean values are plotted as function of the nondimensional concentration $c[\mu]$ in figure (8). It shows that $\Delta(Re.f)$ is not a linear function of concentration (shown by the dashed line in figure (8)) as postulated by Batchelor's model [10] ,
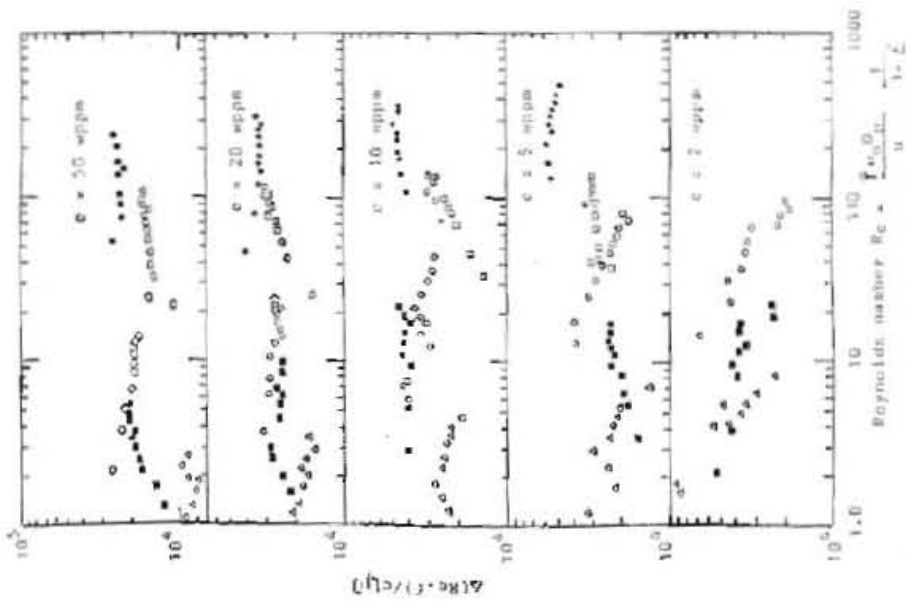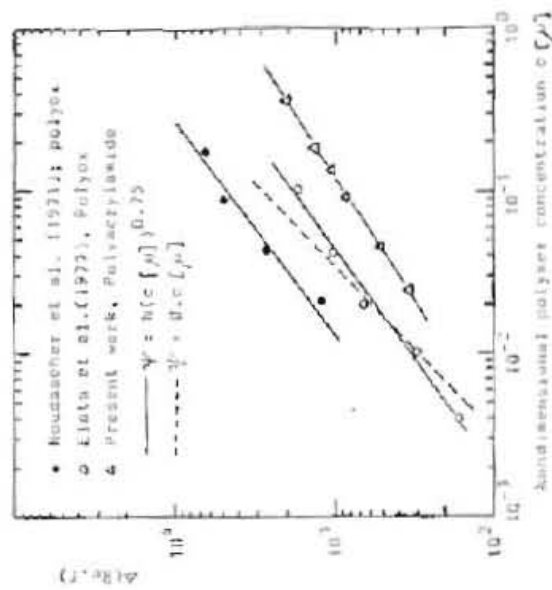
Fig.(18) Increase in flow resistivity $\Delta(Re-f)$ due to polyacrylamide additives as function of the nondimensional concentration $c[\mu]$.

Fig.(17) $\Delta(Re-f)/c[\mu]$ versus Reynolds number for different polyacrylamide concentrations 2,5,10,20 and 50 wppm. respectively.

but, it is best related to $(c[\mu])^n$ where n = 0.75. This does not agree with the value of n = 0.5 reported by Elata et al. [5] and Naudascher et al. [6] for the flow through spherical particles porous medium. Naudascher et al. [6] attributed such lower values of n to shear degradation of polymer molecules as they flow through the porous medium. This is not a reasonable interpretation for the fact that it is a function of the flow rate and not of polymer concentration. Its effect will appear as a decrease in polymer effectiveness with flow rate. It is more logical to be explained on the base of the behaviour of polymer molecules in such flow fields. Polymer molecules can attain their maximum stretching only when they are aligned in the direction of major strain which is almost the direction of the flow. Polymer molecules misaligned that-direction will not stretch enough and sometimes will not stretch at all. These molecules will not contribute in the increase of the extensional viscosity $\eta$ . This is more reasonable to assume that not all polymer molecules are alingned in the major strain driection as assumed by Batchelor's model. Hence, in such flow situation extensional viscosity at saturation condition is best assumed to be proportional to $(c[\mu])^n$ where n < 1.0 rather than to $c[\mu]$.

As shown in figure (8), relation (16-a) can be rewritten as

$$\psi = N (c[\mu])^n \qquad\qquad \dots\dots(18)$$

where N = $1.069 \times 10^4$; n = 0.75

Relation (18) is used with equation (16) to predict the friction factor of porous media dilute polymer solution flows. The results are shown as a solid line in figures (5) and (6) in comparison with experimental data. A good agreement is shown for all polymer concentrations tested. This clearly demonstrate that, within the range of Reynolds number tested, polymers have no influence on the form drag term and the non Newtonian effect is only contributed through increasing the viscous term. Predictions according to relation (16-a) which assumes n = 1.0 are also shown in figures (5) and (6) as dotted line. These results are shown only for comparison.

## 5. CONCLUSIONS

This work presents a study of the flow of dilute solutions of polyacrylamide in a porous medium of non spherical particles. Five porous sections of different particle size ranges are used in order to increase the range of Reynolds number examined. Five dilute polymer concentrations are considered. Experimental results show that very dilute concentrations of linear, high molecular weight polymers as small as 2 wppm cause substantial increase in the flow resistance of porous media when compared with that of the solvent (water). 10 times increase in the friction factor is found to associate the flow of 50 wppm polyacrylamide solution. Such non Newtonian behaviour of dilute polymer solution is attributed to the increase in the elongational viscosity which is found to characterize that type of fluids. Such effects appear when the flow structure is dominated by extentional flow fields as that of porous medium and that of the near wall region of turbulent boundary layer. To that property, drag reduction in turbulent shear flow is attributed by influencing the bursting process of the viscous sublayer as discussed before.

Experimental data, which covered the laminar, transition and
early turbulent flow regimes, show that polymer additives increase
the flow resistance through increased resistance to elongational
deformations. It is sometimes known as the elastic effect and is
similar to viscous drag. The results show also that the increase
in flow resistance due to polymer additives is proportional to
$(c[\mu])^n$ with n = 0.75 rather than n = 1 as postulated by Batchelor's
model. This is attributed to polymer molecules that misaligned the
major strain direction and do not contribute in increasing the elonga-
tional viscosity. Elata et al.[5] and Naudascher et al.[6] reported
lower value of n = 0.5 for the flow of polyox solutions. In fact,
many investigations are still needed to give convincing answers to
the many questions arised.

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